Hydro-Economic Modeling Framework to Address Water-Energy-Environment-Food Nexus Questions at River Basin Scale

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Executive Summary

Increasing competition for water resources among multiple economic and social sectors calls for efficient allocation of water and intelligent trade-offs among sectors. To support such an integrated planning approach, there is a need for tools that better account for the complex dynamics underlying water systems. Hydro-economic modeling is one such tool: It is typically used to understand how the economic benefits from water allocation can be improved or optimized or to assess the economic benefits of policy or infrastructure responses to current and changing conditions. Many hydro-economic models (HEMs) exist to study such problems, but a recent review of these tools points to areas where progress and innovation would improve their relevance. These include improvements in the representation and analysis of feedbacks between water and other systems in the economy (energy and industry, for example), more sophisticated accounting of ecosystem services, as well as analysis of the distributional implications of alternative management institutions (Bekchanov et al., 2017).

The underlying structure of HEMs is node-based, with flow continuity equations describing water movements (natural flows as well as human-controlled supply, storage, and distribution to demand locations) throughout a river system (Harou et al., 2009). This organization is useful for its detailed spatial and temporal representation of water resources systems. Such models are widely used in forecasting and scenario analyses to compare the economic consequences of environmental (e.g. water supply availability), technological (e.g., introducing drip irrigation), infrastructural (e.g., dam/reservoir development), and institutional (e.g., water markets, water pricing, or market liberalization) changes. The HEM framework suggested in this report is largely based on this structure, but places additional emphasis on interlinkages across the Water-Energy-Environment-Food nexus, which increasingly challenges the decisions of water and energy systems managers (McCornick et al., 2008). Nonetheless, it is important to acknowledge that HEMs have often included and considered trade-offs across the production and consumption needs of the energy and agricultural sectors, so our work should be considered an extension, rather than a re-invention, of such models.

The water, energy, and food components in this nexus HEM are controlled by the social system, which itself falls within a larger environmental ecosystem. The social system is comprised of individuals and communities that use water and other resources (e.g., land, energy) as well as the institutions that manage them. Each system also generates externalities, for example pollution, that affect inhabitants of the ecosystem in complex ways. Pollution externalities in particular have an adverse effect on the ecosystem's ability to provide services to the broader system. One central theme of the nexus approach is security, here defined in terms of water, food and energy security. These various notions of security are closely related to the concept of availability, access, and affordability (3As) of essential goods and services (Flatin and Nagothu, 2014). The availability of resources and final services depends upon biophysical conditions, domestic production, and regional trade. Production processes require built and social infrastructure and capacity. Accessibility and affordability, therefore, depends upon existing societal structures such as markets and allocative institutions and upon technological and economic opportunities. To address issues related to 3As, biophysical, economic, and institutional conditions are crucial.

Our HEM Nexus framework depicts interactions between five specific sectors or modules. The first core module is the water system; it is based on the typical node-link structure of most similar HEMs and necessarily contains linkages between surface and groundwater resources. Three other modules that are linked to this core are principally human production systems: energy, municipal and industrial, and agricultural production systems. A fifth module describes the broader ecosystem or environment; this component provides a variety of market and nonmarket goods and services (ecosystem services) to the other systems and is also the recipient of ?externalities? from them. These externalities, beyond certain levels, may lead to a reduction in the ability of ecosystem to provide services to other systems and to the environment.

The structure of the HEM Nexus framework is based on three concepts: scalability, i.e. the HEM should be able to represent basins or regions of different scales; transferability, i.e. the model should be transferable across river basins without substantial effort to change its underlying structure; and modularity, i.e. each module that is connected to the core water system should be able to function independently. The four connected modules are linked to the core via decision variables that enter the model objective function. This objective function aims for maximization of benefits across sectors and uses given both physical and social water and energy system relationships and constraints.

As an optimization model, the HEM Nexus tool is well-adapted to identifying solutions that most efficiently allocate water and other resources, which is especially useful for planning purposes. As with all similar models, it works from a standardized and simplified representation of a very complex system that is developed to be both sufficiently realistic and computationally tractable. Such models are sometimes criticized for the assumptions inherent in their structure. Optimization frameworks for example may not be well-suited to understanding real world outcomes because the institutions governing allocations rarely come close to resembling an omniscient social planner or a well-functioning water market. In addition, the model is not meant to be used for operational purposes, which typically require greater spatial and temporal resolution. A basin scale, node-based HEM framework, as suggested in this paper, works well at basin level and is best suited to answering questions related to investments and policies, water use optimization across sectors, trade-offs across sectors, and connections with ecosystem services. Such an HEM may need to be linked with more detailed economy-wide models to better understand the issues of affordability and accessibility. Finally, the HEM Nexus described here is new and needs to be applied to a variety of problems and contexts to improve its usability and relevance to real world situations.

1 Introduction

Future projections of water supply and demand suggest a trend towards increasing global and regional water scarcity (Rosegrant et al., 2002; Alcamo et al., 2007; Arnell et al., 2011; Hanasaki et al., 2013; Schewe et al., 2014). Reflecting this increased scarcity, analyses of likely future climate and socio-economic change point towards greater competition for water among various sectors of economy as well as the environment (Rijsberman, 2006; Chartres and Sood, 2013; Mancosu et al., 2015). Given this trend towards increased water competition, it will become increasingly crucial for society to efficiently and effectively manage allocations among competing uses. Various institutions will play an important role in this management process; these institutions will need to understand and balance numerous and complex trade-offs across sectors including agriculture, municipal and industrial, hydropower and energy, and environmental and recreation.

A careful balancing of such diverse interests requires that water resource planning continue to evolve from an approach focused on analysis of isolated projects and solutions towards more integrated consideration of development trajectories and portfolios of management and investment solutions. The tools needed for such analysis must achieve increasing integration and flexibility of ideas and principles from both physical and social science disciplines. Much progress has already been made in establishing robust hydro-economic models for use in water resource planning applications (Harou et al., 2009; Bekchanov et al., 2017), but the dominant approach in the field continues to focus on isolated objectives, e.g., maximization of water use benefits in hydropower production and/or irrigation, minimization of municipal and environmental water delivery costs, or management of well-defined risks. A more integrated approach requires that water demands and benefits from multiple sectors and interlinkages among these sectors be considered simultaneously and that trade-offs across them be analyzed to better understand how to efficiently deliver benefits to society as a whole.

The integrated approach to water resource planning first became prominent with the launch of Water Resource System Analysis (WRSA), defined as ?study of water resources systems using mathematical representations of the component processes and interactions of the system to improve understanding or assist in decision making? (Brown et al., 2015). WRSA began with development of a systems approach to water resource planning that included multi-objective optimization of water infrastructure investments (Maass et al., 1962). Since then, it has evolved into a more collaborative analytical approach, whereby stakeholders are involved in defining the relevant systems and couplings between them. The building blocks of the models used for analysis and understanding of interactions and feedbacks consist of mathematical functions that link together hydrological and human components (Brown et al., 2015).

Hydro-economic modeling is one particular WRSA tool that aims to understand the economic implications of interactions between human and water resource systems. Hydro-economic models (HEMs) are typically developed to understand the optimal economic benefits from water allocation or to assess the economic benefits of policy or infrastructure responses to current and changing conditions. The central concept for describing economic value in such models is that of marginal benefit, which is differentiated according to the type of water use. Traditionally, economic analysis using HEMs has been conducted to understand how changes in the availability of water ? from infrastructure, altered management and/or operating rules, or changing flow conditions ? translate into changes in marginal and overall economic benefits (Bekchanov et al., 2017). Thus, water allocation is driven by the value of water with the goal of increasing or maximizing its overall benefit to human society. To achieve optimal economic efficiency, water is allocated among various users until the marginal net benefit across uses is one and the same.

HEMs include spatially and temporally-differentiated data and flow continuity (mass balance) relationships that describe movements of water using a node-link network structure (Brouwer and Hofkes, 2008; Cai, 2008; Harou et al., 2009). Water flows move naturally through the network but can also be modified using potential and existing water management infrastructures. Water is then consumed, subject to its availability, according to the spatial configuration of economic agents and their demands, with infrastructural operating rules and/or allocative institutions acting through a set of constraints or decision variables (Bekchanov et al., 2017). Hydrological flows can be provided as a time series of inputs based on historical conditions in a basin or can be obtained from rainfall-runoff models that allow for consideration of changing climatic conditions. Water management infrastructure includes natural and human-built infrastructure, the latter of which can lead to temporal smoothing of variability in water availability at a particular location. Economic water users are associated with demand functions that both link quantities of allocated water to marginal or total benefits and also encompass nonmarket uses. Finally, management costs include those related to infrastructure development, storage, pumping, transfer, and distribution of water resources.

HEMs are often also distinguished according to whether they are simulation or optimization models (Harou et al., 2009; Bekchanov et al., 2017). Network-based simulation models are widely used in forecasting and scenario analyses to compare the economic consequences of environmental (e.g. water supply availability), technological (e.g., introducing drip irrigation), infrastructural (e.g., dam/reservoir development), and institutional (e.g., water markets, water pricing, or market liberalization) changes. Optimization models on the other hand allow for determination of the most efficient water allocations within a system under varying conditions and subject to a variety of constraints.

When it comes to analysis of the interlinkages between water and economic systems, the usefulness of a particular model structure depends on the research question and objective at hand. A recent review of basin-scale HEMs and economy-wide water models identified a number of critical research gaps that would improve the usefulness of such tools (Bekchanov et al., 2017). One critical shortcoming concerns a lack of sufficiently realistic integration of water, energy, and food systems. A second major gap concerns the often poor representation (and therefore understanding) of the value and systems trade-offs surrounding nonmarket water-related ecosystem services. HEMs by definition include many ecosystem services since these tools describe use of a specific natural resource, water, by a range of sectors. Inclusion of nonmarket water-based provisioning and regulating services, however, is often challenging.[1] Finally, most HEM studies tend to gloss over or oversimplify the importance and consequences of institutional constraints for economic production. Indeed, water allocation decisions are rarely made based on some idealized optimal value of water but, rather, within a complicated context of political and social constraints. As such, institutions can act as facilitators of, or obstacles to, efficient water allocation.

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[1] These services include aspects such as soil fertilization; maintenance of subsistence livelihoods, wetlands and ecological function; pollution and erosion control; and many recreational values.

This paper describes a new and general HEM structure that aims to allow researchers to address some of these gaps. The next section presents a conceptual framework based around the Water-Energy-Environment-Food (WEEF) Nexus concept that helps to organize ideas by illustrating the scope and scale of the challenges facing integrated models of this type. We then review the prior literature that is relevant for understanding the myriad linkages in the WEEF Nexus using HEMs and highlight some of the most critical gaps in this prior work. Section 4 explains how our new HEM structure aims to fill these gaps, using pictorial schematics to clarify which aspects and connections have been included in the model. Section 5 describes the mathematical structure of the model in full detail, beginning with a presentation of the principles that were applied in its creation–scalability, transferability, and modularity–and then proceeding with a listing of equations and definitions for model parameters and variables. Finally, the report concludes by highlighting the importance and shortcomings of this work, suggesting how the model may be usefully applied in future research, and summarizing the lessons learned through this effort.

2 Conceptual Framework

Before developing a conceptual framework for nexus-based hydro-economic modeling, it is important to develop a more precise definition of what we mean by the Water-Energy-Environment-Food (WEEF) nexus. The basic idea motivating use of this nexus concept is that each of these various systems are interlinked and that the interlinkages and feedbacks across them must be considered in holistic fashion if development planning is to be improved. The energy and food systems may be considered as human production systems that influence and are influenced by the constraints and opportunities of the wider social system, all of which also fall within the environmental system (Figure 1). The social system is made up of the individuals and communities that use resources to produce economic benefits as well as the institutions that manage them. It demands resources (e.g., land, water, timber) from the environment, labor inputs from society, and intermediate and final goods from the three human production systems. The quantities of inputs and outputs that are demanded by different stakeholders in the social system depend on demand functions that relate quantities to willingness to pay (or marginal benefits). These marginal benefits are not static; rather, they evolve as a function of technology, demographic and other changes, and societal preferences.

Production is supported by natural resources derived from the ecosystem (a subset of broader ecosystem services), and accessing these resources entails costs that vary over time and space. Each system also generates externalities, for example pollution, that affect the inhabitants and ecology of the ecosystem in complex ways. Pollution externalities in particular have an adverse effect on the ecosystem's ability to provide services within the broader nexus.

Substantial prior work has worked to elucidate the theoretical interdependencies between these WEEF sectors and has correspondingly argued for the importance of an integrated approach to management in these domains (see for example McCornick et al. (2008), Bazilian et al. (2011), Ringler et al. (2013), Arent et al. (2014), Weitz et al. (2014)). Traditionally, nexus discourse has also been driven by a debate over the interrelated components of resource security (Hoff, 2011; ADB, 2013; UNESCAP, 2013; Dubois et al., 2014). The notion of water security for example refers to safeguarding "sustainable access to adequate quantities of acceptable quality water for sustaining livelihoods, human well-being, and socio-economic development, for ensuring protection against water-borne pollution and water-related disasters, and for preserving ecosystems in a climate of peace and political stability" (UN Water, 2013). Food security meanwhile is achieved by ensuring "physical and economic access to sufficient, safe and nutritious food that meets dietary needs and food preferences for an active and healthy life" (FAO). Similarly, energy security can be maintained through "uninterrupted availability of energy sources at an affordable price" (International Energy Agency, 2016).

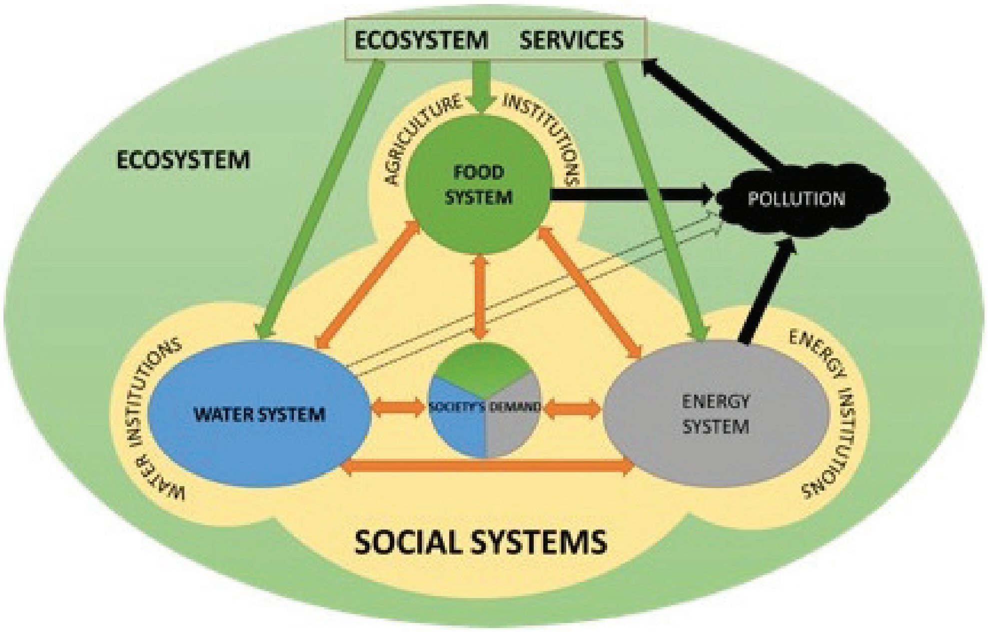

Figure 1: Water-Energy-Environment-Food (WEEF) Nexus framework

The unifying idea in each these definitions is that scarcity–of inputs and resources as well as capacities to use them–can lead to insecurity by threatening access to "sufficient" and "affordable" quantities of water, food, or energy to meet basic human needs. Uninterrupted availability of inputs and resources is thus necessary in securing the ability to achieve economic benefits derived from each of these systems. Nonetheless, resources need not be obtained or produced locally as they can also often be acquired from other regions through trade and migration.

The term ecosystem services then refers specifically to the set of provisioning and regulating features provided by natural resources, including those related to ecological function (Fisher et al., 2009). Many uses of natural resources and other inputs also require complementary inputs of investment or infrastructure development and social capital. Thus, there is often a divergence between potential and economically-relevant resource availability and between potential and actual resource use. When actual availability lies below potential availability due to lack of development, some label the situation as one of economic, rather than physical scarcity (Rijsberman, 2006). This conception of economic scarcity allows for the fact that pure physical availability of resources does not guarantee security. It also accommodates the idea that natural variation in the supply of resources may lead to temporary scarcity in the absence of sufficient investment in infrastructure. Finally, it covers the situation of scarcity that may arise during social disruptions such as economic crisis, famine, war, or sustained institutional failure. In all of these cases, additional investments and trade, better governance, or redistributive policies that help the poor may be required to achieve and maintain long-term security.

In sum, the availability of resources depends upon biophysical conditions, production by domestic or local systems, and regional trade. Use of resources in consumption and production processes require built and social infrastructure and capacity. A lack of security may be created if the cost of utilizing resources for societal needs is too high, either because of high levels of demand or because the cost of exploiting resources is prohibitive to those interests. Accessibility and affordability then depends upon the social structure of the society, stability of markets or allocative institutions, and on consumers' wealth and income. There are no clear boundaries that demarcate availability, access, and affordability (the 3As) (Clover, 2003; Cook and Bakker, 2012), but biophysical, institutional, economic conditions, and institutions clearly play crucial roles. Water can be available but not accessible because of mismanagement or institutional restrictions; it can be accessible but not affordable (such as in case of desalinated water) due to the high cost of technology.

A nexus-based HEM

A review of existing literature on the water-energy-environment-food nexus by UNESCAP (UNESCAP, 2013) shows that the primary issues of concern to researchers in this domain can be broadly grouped under three themes: i) describing the complex inter-relationships between water, energy, and food sectors, ii) the institutional and policy dimensions of these connections, and iii) their broader implications for resource security. As discussed above, one of the primary tools for understanding the physical and economic aspects of water resource systems has traditionally been the HEM. The strength of the HEM as a descriptive and planning tool is its ability to integrate mathematical descriptions of the hydrological (or biophysical) processes that describe water flow with economic production processes that require water inputs and infrastructure investment. Naturally, these production processes already often include energy and agricultural users. Thus, a more flexible, coupled WEEF model should be considered an extension, rather than a re-invention, of the standard HEM framework.

In fact, several systematic reviews of HEM tools indicate surprisingly limited integration–meaning little inclusion of feedbacks–across nexus domains (theme i) as well as a frequent lack of inclusion or realism with regards to institutional constraints (theme ii) (Harou et al., 2009; Bekchanov et al., 2017). This highlights the disconnect between theoretical discourses of the importance of nexus thinking, on the one hand, and the integration of such thinking into practicable and useful decision tools such as HEMs, on the other. These deficiencies hamper utilization of a systematic approach to analyze and understand the implications of nexus policies designed to enhance resource security (theme iii). A fully operational, nexus-based HEM would help in transforming the discourse on the nexus from one based on theoretical interconnections to one aimed at practical and holistic policy making.

A fully-operational nexus-based HEM would closely couple hydrology, energy, and agriculture biophysical models using water as a connecting thread and would enable linking of the biophysical components with economic and institutional realities. If linked to market wide models, such as computable general equilibrium (CGE) models, nexus-based HEMs could also help researchers understand final economic outcomes in terms of income and consumption at the sectoral, community, and/or household levels. The critical first step, however, it to consider the detailed connections and feedbacks between the various production WEEF systems.

Thus, we begin by depicting the interactions between five sectors or domains (Figure 2). Four of these represent human-centered use or production systems (water, energy, municipal and industrial, and agriculture), and the last corresponds to the broader ecosystem or environment. In this conception, the ecosystem domain provides a variety of market and nonmarket goods and services (i.e., ecosystem services) to the other systems and is also the recipient of pollution and other "externalities" from them. These externalities, beyond certain levels, may lead to a reduction in the ability of the ecosystem to provide services to other systems and to the broader environment.

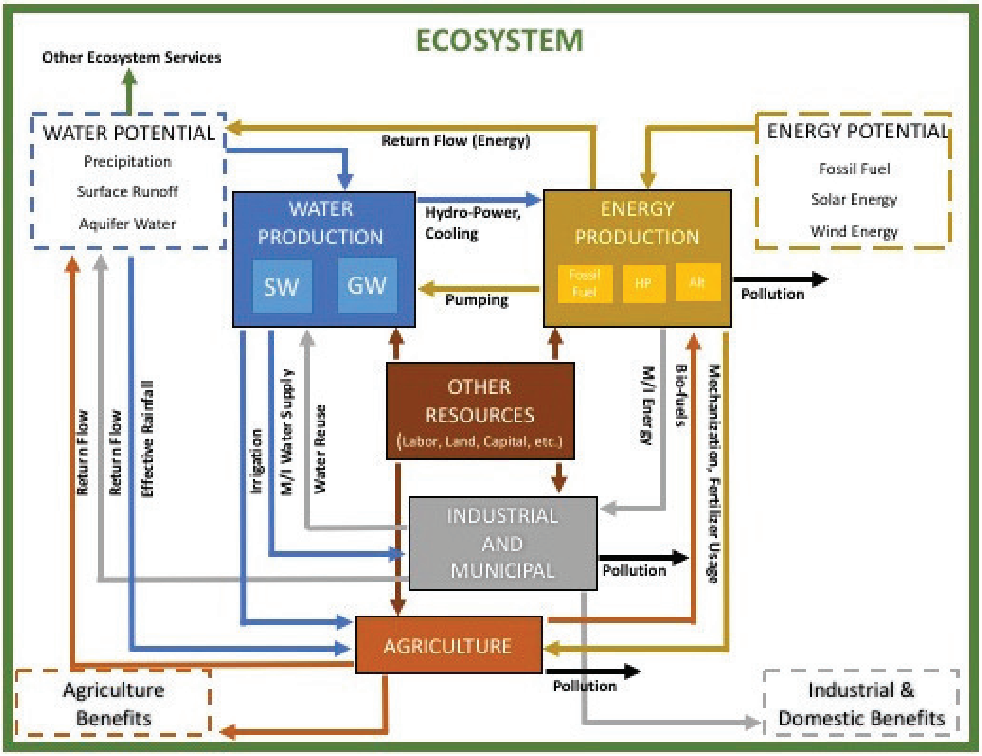

Figure 2: Interactions between production domains included in the WEEF framework

The first production domain represents the water system. Water is an essential natural resource for many economic and environmental functions, and is produced within the ecosystem. Rainfall and glacier or snow melt fills rivers, lakes, and reservoirs through surface runoff, infiltrates into the ground and storage aquifers, and contributes to soil moisture or storage in living plants and animals. Surface water and groundwater resource connections are influenced by physical properties of the local surface and subsurface. Water supply from this system is then allocated into one or more of the other three production systems (energy, industrial and domestic, and agriculture production), or remains in the natural environment, where it plays an essential role in a variety of other regulating and provisioning services. Utilization of these water resources by the three production sectors typically requires intervention and infrastructure. This infrastructure can include storage to cope with spatial and temporal variability in water availability, conveyance that moves water to the point of intended use, or pumping to bring water to the surface or to higher elevations. The flows of water towards human uses are termed water production (WP).

We provide a more detailed schematic of the connections between the WP system and the other 4 domains in Figure 3. The elements of WP can be categorized as being related to supply or demand. On the supply side, water potential is divided into surface and groundwater resources.

These ground and surface waters are connected by hydrological processes such as seepage and infiltration. The potential surface water available in a given location is the water that flows directly from upstream locations plus any surface return flow from other production sectors. The potential groundwater that is available consists of water stored in aquifers plus natural recharge and groundwater recharge from the production sectors.

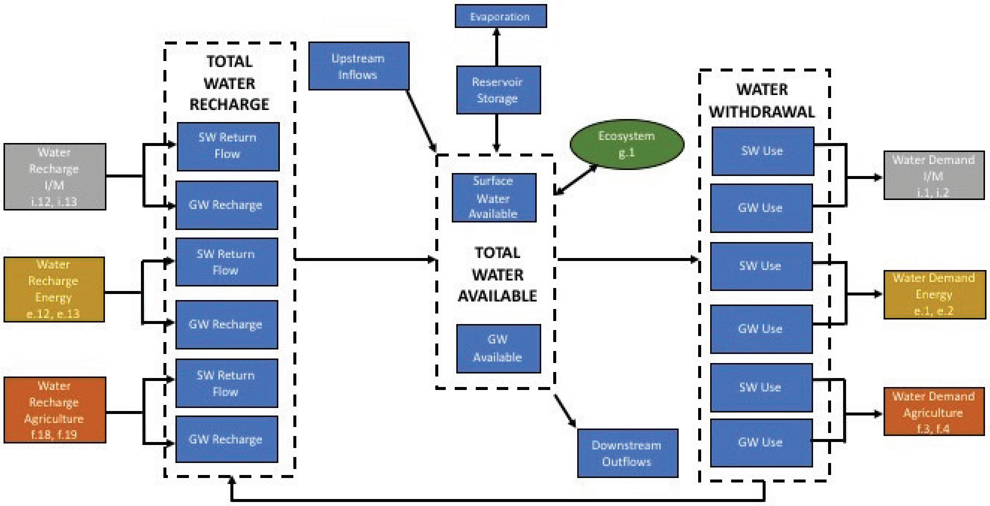

Figure 3: Schematic depiction of the Water Production (WP) system. Attributes or variables that are primarily related to the water system are shown in blue; energy in yellow; municipal/industrial in grey; and environmental in green. Model equations related to the interactions between the WP system and other production systems are included where applicable.

Water potential translates into water availability based on the location and capacity of existing storage, and connectivity of supplies to demand sites using conveyance infrastructure. Surface storage leads to loss of water from the system due to evaporation. Surface water not held in storage, and/or allocated to environmental flows, moves downstream. Actual availability of water for each of the other production systems may also depend on energy supply, which is needed to pump water to end users, especially for groundwater or for conveyance over long distances. The use or exploitation of the water potential will be determined by demand, from industrial/ municipal (I/M), agriculture, and energy sector users. Finally, the broader ecosystem both influences and is influence by the WP system.

Figure 3 relates this construct of the WP system to the HEM developed in Section 5 by including references to the equations that specify the links identified in this figure. Each additional production system (energy, agriculture, and industrial/municipal) links into the WP system through water demand and return flow. The approach is utilized in Figures 4-6 as well to illustrate the relationship between the conceptual construct of the inter-sectoral relationships with the modeling application.

Domain 2, Energy Production (EP), comprises the energy system (a detailed depiction of the connections within this domain and to the other domains is shown in Figure 4). As in the WP system, the ecosystem provides resources to the energy system, including fossil fuels such as oil, coal and gas and renewable energy potential from solar, wind, geothermal, or hydroelectric sources. Exploitation of these energy resources requires processing and infrastructure. Along with other investments, this exploitation necessitates water inputs. In particular, water is used for drilling, by refineries for oil and gas production, for dust suppression and washing in coal production, for irrigation of biofuel crops, for steam generation and cooling in thermal plants, for cooling in nuclear plants, and for hydropower generation. Conversely, energy production affects the quality (through pollution by chemicals or heat) and quantity (based on the balance of evaporation, embedding of water into products, and return flows) of water that can be used for other purposes (IEA, 2012).[2] For example, tapping groundwater resources and water supply conveyance require pumping, creating a potential trade-off between more energy intensive use of proximal and often higher quality resources (from aquifers), and more distant and lower quality sources (from surface water). Because of these connections between energy and water systems, economic water scarcity can arise from either insufficient energy or water supply infrastructure, or both.

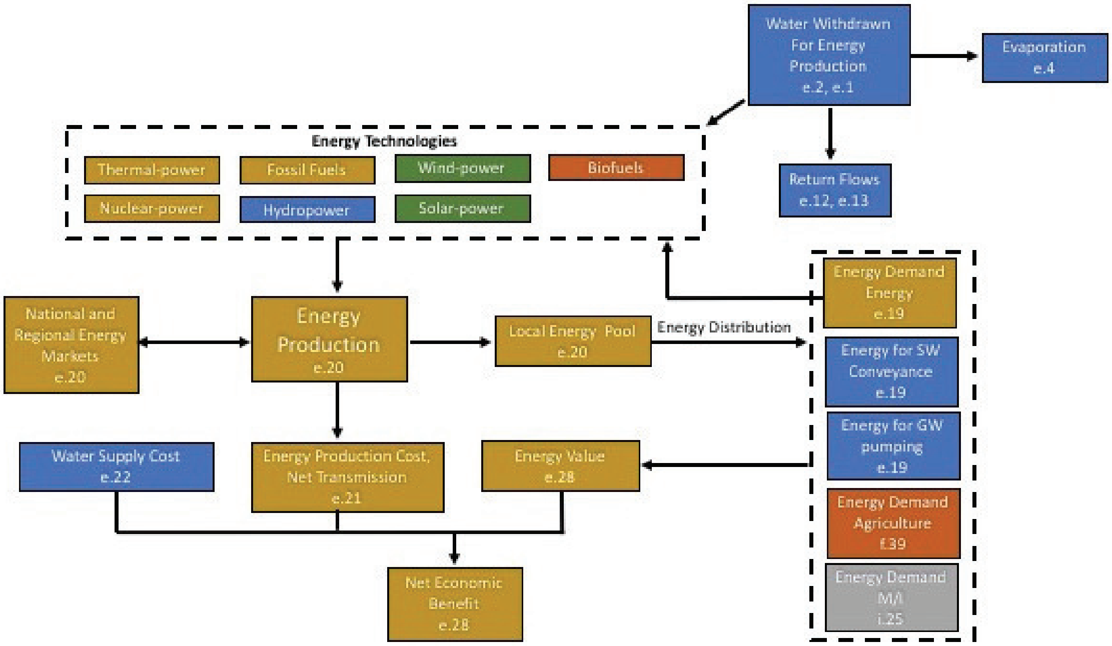

Figure 4: Schematic depiction of the Energy Production (EP) system. Attributes or variables that are primarily related to the water system are shown in blue; energy in yellow; municipal/industrial in grey; and environmental in green. Model equations related to the interactions between the EP system and other production systems are included where applicable.

Central to this domain is the idea of a "National/Regional Energy Pool", which may be considered as analogous to a storage reservoir in the WP domain. Whereas water storage pertains to a catchment, the regional energy pool, which contains locally produced energy as well as imported energy, lies within institutional boundaries. Linkages between the regional energy pool and national (or global) energy markets provide connections across political boundaries. The transmission lines that form these connections are especially important because of the particular challenge of storing energy.

The supply system for energy is broadly characterized into electricity and fuel. Each group can be further subdivided based on specific energy sources to better define the cost of energy within a given region. Electricity can be generated by thermal, nuclear, wind, solar or hydropower.

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[2] In fact, according to IEA analysis, global water withdrawals for energy production in 2010 were 583 km^3, representing about 15% of total global water production, but only 11% of these withdrawals were consumed (i.e. not returned to the environment). Fossil fuel and nuclear power plants were the largest users of water due to the need for cooling water; this emphasizes the importance of return flows (and effects on quality) from this sector.

Fuel is made up of fossil fuels and biofuels. There also exists some overlap between the two; for example, thermal energy can be used in the production of fuels and fossil fuels themselves can generate electricity. The local pool of energy may be augmented by production of energy within municipalities (from waste) and industries (for their internal use), which is usually consumed locally by the M/I sector. In analyzing the true economic costs of different energy sectors, it is important to note that actual cost to users is frequently distorted by regional policies (price controls or subsidies) and that related adjustments are necessary to complete accurate analysis.

Water demand by local energy producers can be estimated as a function of energy generation, and is linked to the production systems in the WP domain. The flow balance that determines the balance of consumption, losses, and return flows to the WP system closes the loop. As in the case of WP domain, the energy available in the regional energy pool is distributed to the other domains (water, municipal/industrial, and agriculture) according to an economic objective (maximizing net benefits) or according to specific allocation rules and regulations.

The third domain, Agriculture Production (AG), concerns food production system (Figure 5 presents its schematic, again with connections to the other domains). Throughout the world, the agriculture sector is typically the largest user of water (representing around 70% of global water withdrawals), and it also often consumes significant energy resources (United Nations, 2016). The purpose of water allocation to this domain is to enable crop irrigation, which improves yields by enhancing control over essential water inputs, protects against droughts, provides production in areas with insufficient rainfall, and allows for higher cropping intensity than rainfed irrigation. Irrigation technology and techniques vary greatly, influenced by infrastructure investment on large (e.g., canals) and small (e.g., field technologies such as drip vs. spray) scales. This leads to different levels of water use efficiency across irrigated areas.

In low efficiency systems, less water is effectively used by crops, and more water evaporates and drains back into ground or surface water bodies, along with pollutants such as pesticides and fertilizers. In contrast, higher efficiency systems have higher rates of consumption, and lower return flows. These efficiency differences translate into varying patterns of energy consumption, due to differences in pumping requirements (which are usually higher for low efficiency systems because more water must be pumped) or technology. The agriculture sector, meanwhile, requires energy for other activities in addition to irrigation, including mechanization and fertilizer usage. Agriculture is not only a user of energy, however; an important feedback loop comes from its contribution to the energy system through biofuel production. Biofuels include a range of products (such as bio-alcohol, ethanol, bio-diesel etc.) that are made from crop-based sugar, starch, and vegetable oils.

The crops considered in the AG module are classified as rainfed and irrigated. Rainfed crops get their water only through precipitation (or effective rainfall, which refers to the fraction of rainfall used by crops). For irrigated crops, effective rainfall is augmented with allocations from surface or groundwater supplies. Each crop requires a specific amount of water to reach maximum yield in a particular region. Deviations from this requirement lead to water stress and crop-specific reductions in yield. The product of area under cultivation and yield then gives the total production of crops in the region. Energy is required for conveyance of surface water and pumping of groundwater; its cost depends upon distance conveyed, as well as depth and pumping technology (capacity and pump efficiency). This and other inputs in the agriculture sector (e.g., labor, fertilizer, etc.) also influence crop yields and production. Net profits for producers then come from the difference between revenues (or prices multiplied by production) and these various input costs.

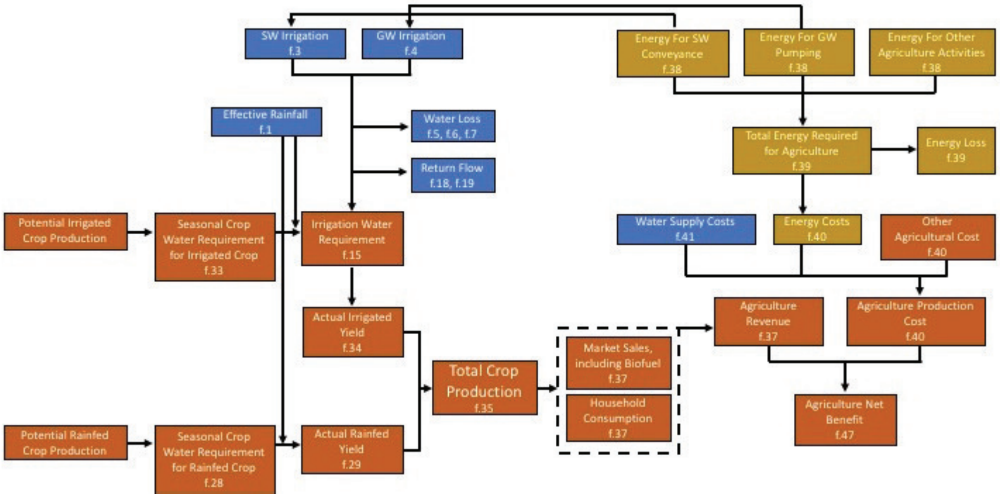

Figure 5: Schematic depiction of the Agricultural (AG) system. Attributes or variables that are primarily related to the water system are shown in blue; energy in yellow; municipal/industrial in grey; and environmental in green. Model equations related to the interactions between the AG system and other production systems are included where applicable.

The fourth domain, Municipal and Industrial (MI), represents consumption of water, food, and energy by humans for domestic purposes and for the production of industrial consumer goods (Figure 6). Households demand food and water to meet their dietary needs and maintain good health, demand water for other domestic purposes (cooking, hygiene, etc.), and demand energy for lighting, cooking, and heating. Yet there are wide disparities in water, food, and energy consumption across the globe, which are correlated with infrastructural and institutional capacities to tap water and energy resources, as well as with regional preferences and conditions and socio-economic factors. Production of water for domestic purposes also requires energy to enable effective drinking water treatment and distribution to users. In addition to domestic requirements, water and energy also factor into the production of intermediate and final consumer goods. In fact, the industrial sector is the second largest global consumer of water and the largest consumer of energy (United Nations, 2016; U.S. Energy Information Association, 2016). Water usage by households and industry also generates substantial amounts of polluted wastewater, which may or may not be treated prior to its discharge back into the environment depending on energy availability and infrastructure.

Water and energy demand depend on socio-economic factors such as population, per capita GDP, and urbanization. Furthermore, these demands provide the links between the MI domain and the WP and EP domains, and consumption of these inputs arises again from the profit maximizing behavior of firms in the sector and utility-maximizing behavior of consumers. Specifically, firms balance input costs for water pumping, treatment, and distribution along with the cost of energy purchases with the revenues derived from production of industrial goods. Usage of water and energy within this sector entails losses from evaporation during conveyance as well as in distribution and transmission of electricity. Some water may be reused after adequate treatment, and waste generated in the M/I sector may be used to generate energy for local consumption. Meanwhile, municipal distribution of water and energy services aims to satisfy consumer demand for energy and water, often by institutionalizing cost recovery pricing. Benefits in this domain thus arise from consumer surplus and the producer and consumer surplus produced by the industrial sector.

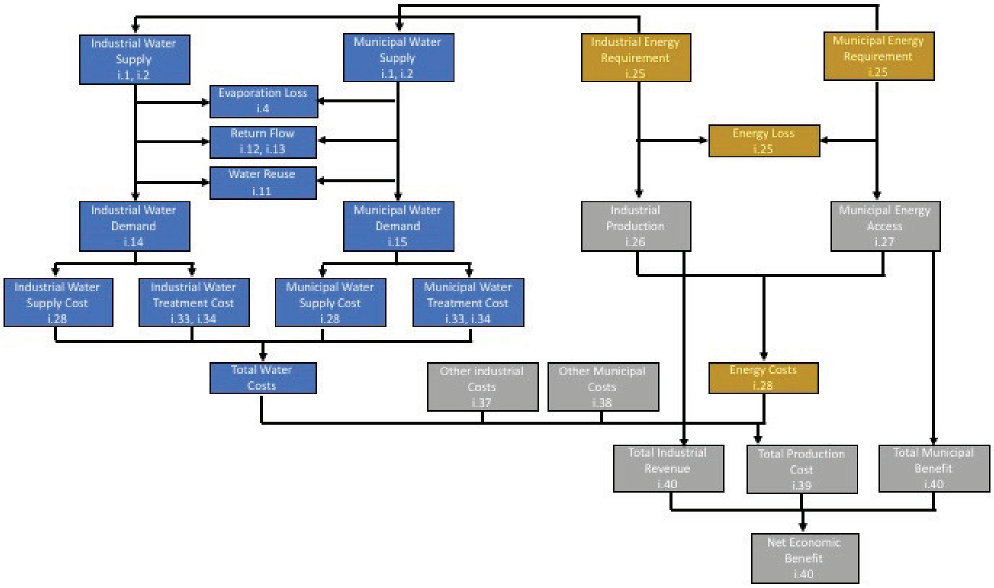

Figure 6: Schematic depiction of the Municipal/Industrial (MI) system. Attributes or variables that are primarily related to the water system are shown in blue; energy in yellow; municipal/industrial in grey; and environmental in green. Model equations related to the interactions between the MI system and other production systems are included where applicable.

All four domains discussed thus far connect back to the Ecosystem (ES) domain (Figure 7). The production of other services (not depicted in the systems described above) from the ES domain–such as fisheries, recreational values, disaster risk mitigation, existence values, etc.–depends on the temporal and spatial distribution of water availability and quality. Water quantity relates to hydrological variability and upstream consumptive uses by the four production systems. Quality, meanwhile, is influenced by utilization and return flows (which may or may not be subjected to treatment) from these production sectors and by the pollutants released from each sector. The economic benefits from ecosystem services then depend on market or nonmarket values for other provisioning and regulating ecosystem services.

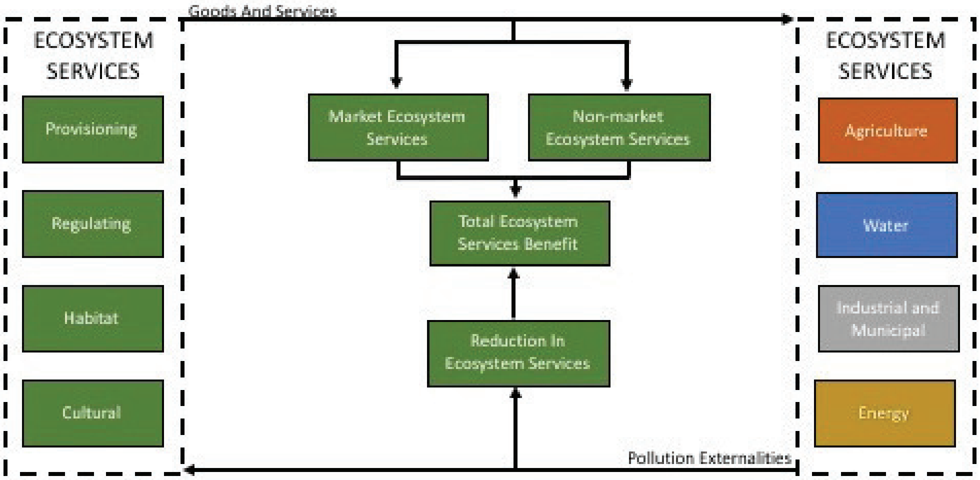

Figure 7: Schematic depiction of the Ecosystem (ES) system. Attributes or variables that are primarily related to the water system are shown in blue; energy in yellow; municipal/industrial in gray, and environmental in green.

3 Relevant Literature

Before presenting the details of the HEM model developed to span across these WEEF nexus domains, we provide a brief review of the literature related to prior hydro-economic modeling efforts to incorporate its different components. This helps to highlight some of the gaps that we aim to fill by developing a more complete integration across these domains and informs the eventual analytical approach we adopt.

In characterizing this literature, a recent and detailed systematic review of water-economy modeling applications that discusses HEMs is particularly helpful. Bekchanov et al. (2017) show that HEMs have been extensively used to analyze the linkages between water systems and the demand sectors described above (i.e., hydropower, agriculture, and municipal/industrial). Many of these prior studies face specific challenges, the most important of which are documented in existing reviews of hydro-economic modeling methods (Brouwer and Hofkes, 2008; Harou et al., 2009).[3] Focused attention on feedbacks to the water system and on cross-sectoral interactions poses perhaps an even greater challenge, in part because it is increasing in importance as population pressure and resource scarcity increase. Most existing multi-sectoral HEM studies consider trade-offs between sectors–predominantly comparing the benefits of irrigated agriculture versus hydropower production (Chatterjee et al., 1998; Barbier, 2003; Hurford and Harou, 2014; Bekchanov et al., 2015) or irrigated agriculture versus ecosystem preservation (Cai et al., 2003; Ward and Booker, 2003; Mainuddin et al., 2007; Blanco-Gutiérrez et al., 2013; Mullick et al., 2013). A small number of notable exceptions consider important system feedbacks such as the demand pressure on water systems that stems from energy use in surface water conveyance and groundwater pumping (Pulido-Velázquez et al., 2006; Harou and Lund, 2008; Kahil et al., 2016) or consumptive water use in biofuel production (Alcoforado de Moraes et al., 2009). A limited body of research examines temporal trade-offs between water use for hydropower production and for dilution of municipal and industrial pollution, usually on a very local scale (such as Yoon et al. (2015)). Out of a total of 160 applications reviewed in Bekchanov et al. (2017), only four focused primarily on trade-offs across WEEF sectors.

Reviews of existing literature also reveal that most nexus-based integrated models are purely bio-physical (Alcamo et al., 2007; Van Vliet et al., 2012; Hanasaki et al., 2013; Miara and Vörösmarty, 2013; Wada et al., 2013). These models typically start from hydrological models that link to sectoral water use models but allocations from them are usually not based on economic principles. Howells et al. (2013), in contrast, developed an integrated application linking climate, land, energy, and water use systems (CLEWS) in Mauritius. CLEWS is an energy focused simulation model that links off-the-shelf models–the Long-range Energy Alternatives Planning (LEAP) model for energy, the Water Evaluation and Planning System (WEAP) mode for water, and an Agro-Ecological Zones land production planning model (AEZ) for land, with climate models (Welsch et al., 2014). The integration of these models to consider sectoral interactions and feedbacks generated significant added value in the test application by highlighting the important effects of water stress on energy production, which led to overestimation of the benefits of ethanol-based energy generation in disaggregated models.

The inclusion of ecosystem services in HEMs remains a major challenge. Ecosystem services

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[3] Prominent among these challenges are the following: a) the need for econometric analysis to evaluate marginal benefit, due to the price distortions that prevail in most water markets; b) the challenge of aggregating demands across different types of consumers or users; and c) the lack of volumetric consumption data in many uses (notably irrigation).

have been broadly grouped into four classes: provisioning, including food production and energy and water consumption; regulating, which deals with controlling climate and diseases as well as pollution control by dilution; supporting, such as nutrient cycling; and cultural, such as spiritual and recreational benefits (Millennium Ecosystem Assessment, 2005). Given the more straightforward connection between provisioning services and economic values, most HEM studies have focused on marketed provisioning services such as water allocation for irrigation or to municipal users. For nonmarket environmental services, the most common approach is to measure trade-offs between market benefits and environmental flow requirements (Bekchanov et al., 2017). Such studies optimize benefits subject to varying levels of environmental flow (or instream flow) constraints. Mainuddin et al. (2007) for example considered how optimized water use in irrigated agriculture changed subject to within- and cross-catchment water sharing constraints. Blanco-Gutiérrez et al. (2013) similarly used an HEM to analyze the loss to agriculture from maintaining environmental flows. Ward and Booker (2003) calculated the economic cost to the agriculture and the municipal and industrial sectors associated with increasing instream flows to meet the ecological needs of a particular fish species in a river system.

A different approach, utilized by Mullick et al. (2013), is the direct estimation the value of ecosystem service benefits. These authors used a hydrologic-economic optimization model to calculate the economic trade-offs between off stream water use (irrigation) and instream water use for fisheries and navigation, using marginal benefit functions that were created for off-stream and instream water use. Cai et al. (2003) include irrigation-induced soil salinization (a regulating ecosystem service) within an HEM analysis of the economic and environmental costs of various irrigation policy options. Ringler and Cai (2006) explicitly modelled water values for wetlands and fisheries in their Mekong River Basin HEM analysis. These direct valuation approaches more readily reveal trade-offs across sectors and uses but require careful derivation of nonmarket valuation estimates for marginal benefits. Nonetheless, a complete nexus approach that considers pollution and return flows must somehow address all such issues.

Finally, it is important to note that many WEEF nexus processes play out on a different and much longer time scale from that governing market processes that evolve via complex dynamics that may be highly nonlinear, emergent, context-specific, and uncertain (Liu et al., 2007). Ecosystems services production has been shown to have these types of features, which tend to challenge existing modeling efforts. The institutions that govern water allocations are similarly lumpy and discontinuous. For example, water sharing treaties with in-stream requirements, as included by Mainuddin et al. (2007), may specify complicated water sharing provisions or constraints on water withdrawals (Mullick et al., 2013; Blanco-Gutiérrez et al., 2013; Ringler and Cai, 2006). Analogous institutions in other sectors–such as energy and agriculture–are rarely if ever included. In a comprehensive nexus-based HEM, constraints in these other domains, such as bio-fuel regulations, renewable energy quotas, water-reuse standards, rainwater harvesting regulations, and cross-sector institutional interactions, need to be considered. This requires careful and detailed institutional mapping across nexus systems, highlighting a potential conflict between generalizability–which is enhanced by accuracy in the description of fundamental processes–and utility for policy making–which stems from well-calibrated and institutionally realistic descriptions that may not reflect fundamental socio-hydrological processes (Beck, 2014).

4 Model Analytical Framework

This section describes the analytical framework for an HEM developed to consider the interconnections in the WEEF nexus framework. We begin by describing the principles applied in the development of our model and then proceed with presentation of diagrams that show how the model relates to the schematics of the broader WEEF concept. This helps to clarify what is and is not included in our formulation. The model equations and definition of variables and parameters follows in Section 5.

4.1 Principles behind the model

The model is developed around three principles aimed at improving the versatility of the final HEM:

1. **Scalability:** The HEM should be able to represent basins or regions (and relevant subunits therein) of different scales and overlap. WEEF nexus issues vary according to the scale of the study area. For example, a small catchment may be dominated by rural populations engaged primarily in agriculture with little energy production or industrial activity or, alternatively, by a single urban setting that includes little to no agriculture. In contrast, a larger scale will likely require inclusion of both rural and urban areas. Also, a smaller area may be dominated by a single institution while larger systems may include multiple institutions. Finally, a scalable model should allow analysis at multiple time scales–for example a single year (as static) or across multiple years–or allow analysis over spatial units of different types such as catchment or geopolitical boundaries.
2. **Transferability:** The model should be easily transferable to any water resource system. This would require that the fundamental structure of the model need not change for different study areas. Differences and idiosyncratic characteristics of a study area instead would be reflected through differences in data.
3. **Modularity:** Outside of the core (which specifies the objective function, the water system, and indicates the other systems included), each module within the HEM framework should be able to function independently. This makes it easier to replace an existing module with an improved version or to "shut-off" modules that are not required to answer particular policy or research questions. It also allows testing of the sensitivity of results that do and do not include integration of multiple sectors, which is an interesting socio-hydrological research question in its own right.

4.2 Schematic presentation of the model

Each of the domains described in Section 2 is represented by a module. As alluded to above, the core is the Water System Module (WSM). This module handles the flow continuity equations that maintain the water balance throughout the system, describes storage in natural and built reservoirs as well as in groundwater aquifers, and specifies water flows in and out of the other production systems or sectors. This core, therefore, contains the objective function that drives water allocations in order to maximize net benefits across domains. The input data into the WSM consists of hydrological inputs (specifically partitioning of rainfall into runoff into surface water nodes and aquifer recharge). These data are best obtained from a separate hydrological rainfall-runoff model that is not directly connected to the HEM.[4] The four other modules that are connected to the core are the Energy, Agriculture, Municipal and Industrial, and Environmental Modules (Figure 8).

Equations pertaining to production processes in each of the other modules are then written within those modules. These are linked to the core via the decision variables that enter the model objective function, and via binary parameters that allow the user to switch the modules on and off. Various additional water and production system constraints appear in the WSM and in the production modules to reflect physical, technological, economic, or institutional realities.

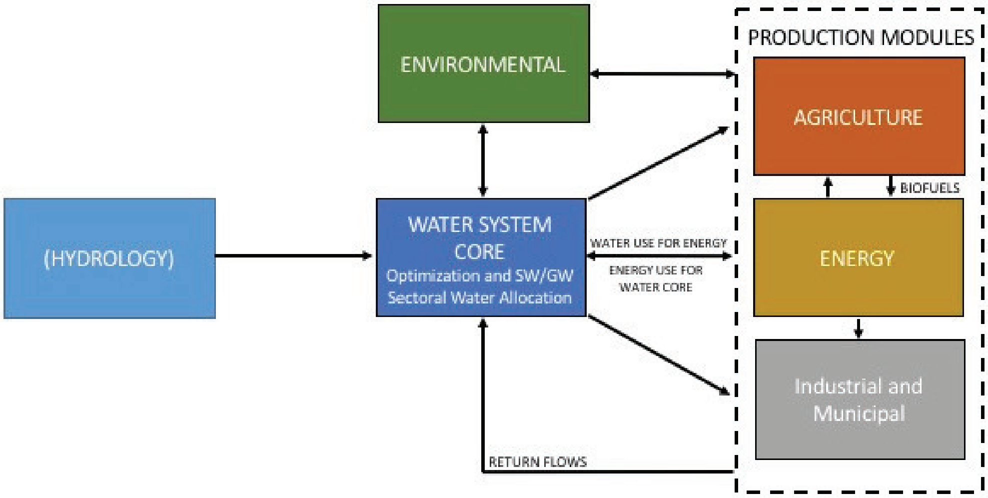

Figure 8: Module interconnections for HEM model

Considering the interdependence of users and the inherent directionality in water resource systems (Keller, 1996; Ringler et al., 2004), integrated WEEF system management is best considered at basin scale. The challenge is then to link basin scale hydrology to policy making in other sectors, given that those decisions are typically made according to a different set of administrative boundaries. Figure 9 shows an illustrative node structure that does not overlap cleanly with institutional (or administrative) boundaries. Reservoirs and/or water withdrawal are represented by "river or reservoir" nodes connected by the flow of a river (links) and into groundwater reservoir nodes. These nodes link to the outlets of the catchments in the hydrological model and represent the physical hydrology of the region. Each node has a surface water and a groundwater component. The surface water component represents the surface water flowing from upstream node as well as the surface water generated within the catchment of the node. The groundwater component represents the groundwater available within the node?s catchment. These are indexed according to institutional boundaries. Production sectors that fall within the institutional boundary but are outside the basin boundary are not considered (as shown by the blackened portion in the figure).

The WSM is then developed around the network of these nodes and links to specify water flow and distribution to users along the river. Economic sectors (or water users) along the river are represented by irrigation, industrial-municipal, and power generation sub-nodes, each of which are connected to parent river or reservoir nodes. Economic sectors also return a fraction of the

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[4] The advantage of this approach is that it allows for the use of previously established and tested process-based hydrological models that incorporate catchment-level complexity and dynamics. Such models readily provide volumes of water stored as soil moisture, groundwater recharge, surface runoff, and water lost to evapotranspiration.

flows they receive to downstream nodes in the surface and groundwater systems through drainage or wastewater flows (return flows). Environmental sub-nodes represent the ecosystem services produced within the catchment represented by each node.

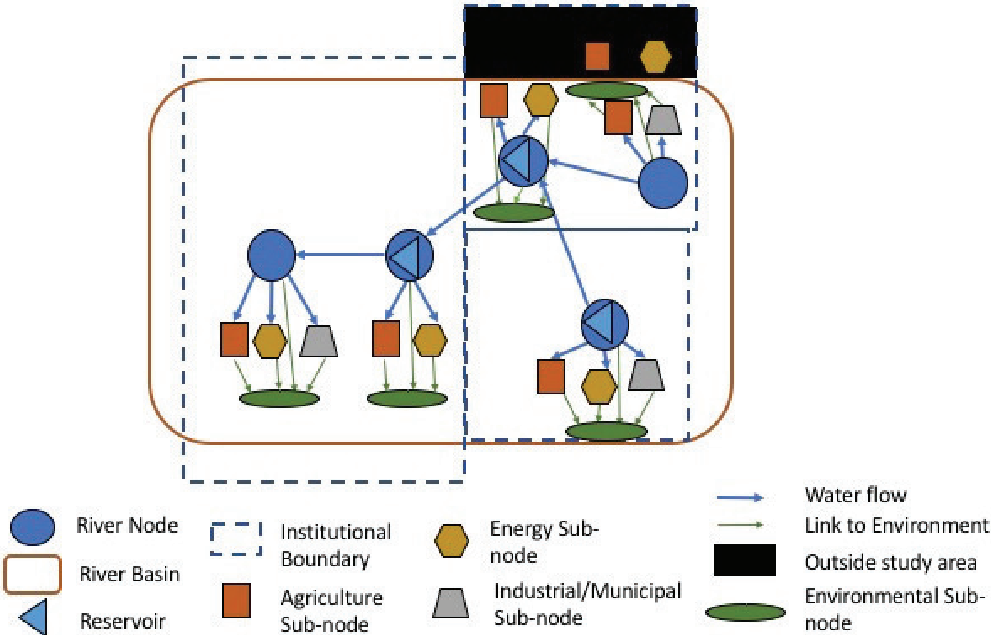

Figure 9: River node network scheme

5 Model Equations

This section presents the mathematical equations that comprise the model. We present these equations by module, and supplement them with diagrams insofar as the latter help to clarify complex relationships between variables.

5.1 Water system module (the core module)

5.1.1 Model objective function

Joint maximization of benefits (B^{OBJ}) across sites n and sectors s is formulated as:

$$B^{OBJ} = \sum_{n}(\sum_{s}\delta^{S}_{n,s}B^{PRD}_{n,s} + \delta^{ENV}_{n}B^{ENV}_{n}) \tag{w.1}$$

where $\delta^{S}_{n,s}$ is a binary parameter that takes a value of 1 if production related to sector s uses water from node n or a value of 0 otherwise;[5]
δ^{ENV}_{n} is a binary parameter that takes a value of 1 if environmental services rely on water from node n and takes a value of 0 otherwise;
$B^{PRD}_{n,s}$ represents the benefit in each production sector that withdraws water from node n; and

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[5] In the GAMS code, such binary indicators are replaced by inclusion of sets that include only the subgroups of nodes pertaining to those sectors.

B_n^{ENV} is the benefit from environmental flows.

As described previously, the main sectors considered in the model are agriculture (A or $sa \subset s$), energy production (E or $se \subset s$), and the municipal and industrial sector (I or $si \subset s$). Separate sets are defined for agricultural (da), energy production (de), and municipal-industrial (di) sites. Thus, only a single sector can be referenced to one node but multiple production sites may belong to this sector according to the model formulation. This notation for sectors and production sites is introduced to make each module independent and to prevent errors in coding. If a particular module is not included in any given application, all binary indicators for that sector (or for environmental flows) can be set to zero using a single input command.

To represent optimization at the institutional level, an institution-specific grouping of nodes can be assigned a differential weighting (according to power or locational asymmetries), or the single global optimization procedure can be broken into sequential optimization problems that begin with the upstream groupings and then proceeds downstream, taking the upstream solution as given when solving the downstream optimization problem (Jeuland et al., 2014).

5.1.2 Surface water balance

Reservoir volume in period $t > 1$ depends on the volume in period $t-1$ as well as the change between periods:

$$V_{r,t}^{W\_RES} = V_{r,t-1}^{W\_RES} + \delta_{r,t}^{W\_V\_RES} \tag{w.2}$$

where:
$V_{r,t}^{W\_RES}$ is the volume of reservoir r in time t;
$V_{r,t-1}^{W\_RES}$ is the volume of reservoir r in time $t-1$; and
$\delta_{r,t}^{W\_V\_RES}$ is the change in reservoir storage of reservoir r in time t.

Reservoir volume in period $t = 1$ ($RES_{r,IVL}^{B\_CHAR}$) is set to an initial reservoir level chosen by the user, and the final reservoir volume must also equal this initial volume (to prevent derivation of unsustainable solutions).

For reservoir nodes, the storage volume and surface area of the reservoir are related to each other using a polynomial relationship:

$$\begin{aligned} A_{r,t}^{W\_RES} = RES_{r,VB0}^{W\_CHAR} &+ RES_{r,VB1}^{W\_CHAR} V_{r,t}^{W\_RES} \\ &+ RES_{r,VB2}^{W\_CHAR} [V_{r,t}^{W\_RES}]^2 + RES_{r,VB3}^{W\_CHAR} [V_{r,t}^{W\_RES}]^3 \end{aligned} \tag{w.3}$$

where:
$A_{r,t}^{W\_RES}$ is the surface area of reservoir r at time t; and
$RES_{r,VB0}^{W\_CHAR}$, $RES_{r,VB1}^{W\_CHAR}$, $RES_{r,VB2}^{W\_CHAR}$, and $RES_{r,VB3}^{W\_CHAR}$ are the parameters of the function that are obtained using regression techniques specific to reservoir r.

If data are missing for particular reservoir sites, a linear relationship between area and volume (and net head and volume, see below) should be assumed as a first-order approximation.

The reservoir net head also depends on the reservoir storage volume:

$$H_{r,t}^{W\_RES} = RES_{r,HT0}^{W\_CHAR} + RES_{r,VA0}^{W\_CHAR} + RES_{r,VA1}^{W\_CHAR} V_{n,t}^{W\_RES} + RES_{r,VA2}^{W\_CHAR} [V_{n,t}^{W\_RES}]^2 \tag{w.4}$$

where:
$RES_{r,VA0}^{W\_CHAR}$, $RES_{r,VA1}^{W\_CHAR}$, and $RES_{r,VA2}^{W\_CHAR}$ are parameters of a function as obtained using regression techniques for reservoir r;
$H_{r,t}^{W\_RES}$ is water level for reservoir r in time t; and
$RES_{r,HT0}^{W\_CHAR}$ is the tailwater level for the turbine discharge for reservoir r.

5.1.3 Node/reservoir water balance

The water balance at the river nodes of the model requires that all inflows to the node equal outflows from it (Figure 10). Water inflows are from upstream nodes, from surface runoff generated within the catchment of the node, from groundwater contribution into the surface water system, and from return flows from production sites. Water outflows are to downstream nodes, to irrigation and municipal and industrial users, water lost due to evaporation, and into groundwater systems. For reservoir nodes, changes in storage are also included.

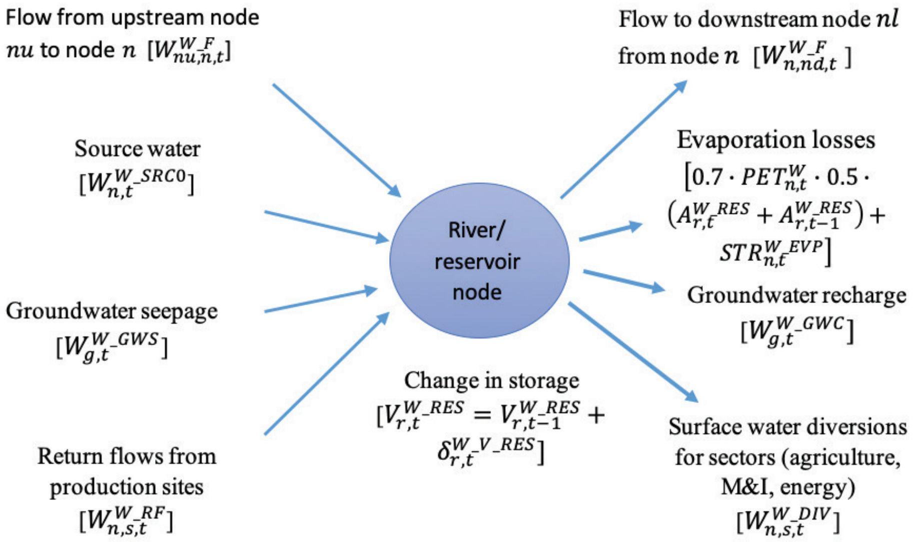

Figure 10: River node/reservoir water balance

Following this logic, the water balance at each river node is formulated as:

$$
\begin{aligned}
&\sum_{nu \in NNULINK} (W_{nu,n,t}^{W\_F}) + W_{n,t}^{W\_SRC0} + \sum_{g \in NGLINK} (W_{g,t}^{W\_GWS}) + \sum_{s} (W_{n,s,t}^{W\_RF}) \\
&\quad = \sum_{r \in NRLINK} (0.7 \cdot PET_{n,t}^{W} \cdot 0.5 \cdot (A_{r,t}^{W\_RES} + A_{r,t-1}^{W\_RES})) \\
&\quad + STR_{n,t}^{W\_EVP} + \sum_{g \in NGLINK} (W_{g,t}^{W\_GWC}) + \sum_{s} (W_{n,s,t}^{W\_DIV}) \\
&\quad + \sum_{nd \in NNULINK} (W_{n,nd,t}^{W\_F}) + \sum_{r \in NRLINK} (\delta_{r,t}^{W\_V\_RES})
\end{aligned}
\tag{w.5}
$$

where:
$W_{nu,n,t}^{W\_F}$ is the flow from upstream node nu to node n at time t (given a link $(nu, n) \in NNULINK$);
$W_{n,t}^{W\_SRC0}$ is the flow from source node (runoff into the river) at time t;
$W_{g,t}^{W\_GWS}$ is the groundwater seepage from groundwater aquifer g at time t (given a link $(g, n) \in NGLINK$);
$W_{n,s,t}^{W\_RF}$ is the return flow to node n, from sector s, at time t;
$PET_{n,t}^{W}$ is the potential evapotranspiration at node n at time t;
$STR_{n,t}^{W\_EVP}$ is evaporation from streams at node n at time t;[6]
$W_{g,t}^{W\_GWC}$ is the water lost to groundwater aquifer g from the river at time t;
$W_{n,s,t}^{W\_DIV}$ is the water diverted from node n, for sector s, at time t; and
$W_{n,nd,t}^{W\_F}$ is the flow from node n, to downstream node nd, at time t (given a link $(nu, n) \in NNULINK$).

5.1.4 Groundwater balance

Similar to the surface water balance in river and reservoir nodes, the groundwater balance requires equality of total inflows and outflows plus water volume change in the aquifer (Figure 11).

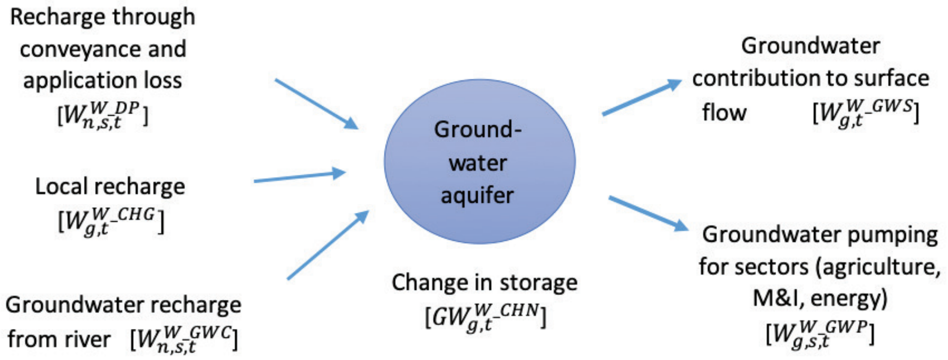

Figure 11: Groundwater aquifer water balance

Groundwater volumes change depending on water percolation from production sites, fields and irrigation canals, groundwater use and water seepage to (and from) the river.

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[6] In the GAMS model, the hydrology input takes evaporation from streams into account, so this parameter is set to 0.

$$\begin{aligned}\sum_{g \in NGLINK} (W^{W\_CHG}_{g,t}) + \sum_{s} \sum_{g \in NGLINK} (W^{W\_DP}_{n,s,t}) + \sum_{g \in NGLINK} (W^{W\_GWC}_{g,t}) \\ = \sum_{g \in NGLINK} (W^{W\_GWS}_{g,t}) + \sum_{g \in NGLINK} \sum_{s} (W^{W\_GWP}_{g,s,t}) + \sum_{g \in NGLINK} (GW^{W\_CHN}_{g,t})\end{aligned} \tag{w.6}$$

where:
$W^{W\_CHG}_{g,t}$ is the groundwater recharge from rainfall at groundwater aquifer g at time t;
$W^{W\_DP}_{n,s,t}$ is the recharge through conveyance at node n, from sector s, at time t (given a link $(n, g) \in GNLINK$);
$W^{W\_GWP}_{g,s,t}$ is the groundwater pumping at groundwater aquifer g, for sector s, at time t; and
$GW^{W\_CHN}_{g,t}$ is the change in aquifer storage for groundwater aquifer g ta time t.

The water table depth from ground surface in period $t > 1$ depends on the depth in period $t - 1$ as well as the change in depth:

$$GW^{W\_D}_{g,t} = GW^{W\_D}_{g,t-1} + \frac{GW^{W\_CHN}_{g,t}}{AQ^{B\_CHAR}_{g,SPY} \cdot AQ^{B\_CHAR}_{g,EAR}} \tag{w.7}$$

where:
$GW^{W\_D}_{g,t}$ is the water table depth from ground surface at groundwater aquifer g at time t;
$GW^{W\_D}_{g,t-1}$ is the water table depth from ground surface at groundwater aquifer g at time $t - 1$;
$AQ^{B\_CHAR}_{g,SPY}$ is the specific yield of groundwater aquifer g; and
$AQ^{B\_CHAR}_{g,EAR}$ is the effective aquifer area of groundwater aquifer g.

5.1.5 Constraints

Maximum and minimum water levels and storage volumes in reservoirs are imposed based on their capacity and minimum operating levels:

$$H^{W\_RES.lo}_{r,t} = RES^{B\_CHAR}_{r,HLO} \tag{w.8a}$$

$$H^{W\_RES.up}_{r,t} = RES^{B\_CHAR}_{r,HHI} \tag{w.8b}$$

where:
$H^{W\_RES.lo}_{r,t}$ is the lower bound of height of reservoir r at time t;
$RES^{B\_CHAR}_{r,HLO}$ is the minimum height of reservoir r;
$H^{W\_RES.up}_{r,t}$ is the upper bound of height of reservoir r at time t; and
$RES^{B\_CHAR}_{r,HHI}$ is the maximum height of reservoir r.

$$V^{W\_RES.lo}_{r,t} = RES^{B\_CHAR}_{r,VLO} \tag{w.9a}$$

$$V^{W\_RES.up}_{r,t} = RES^{B\_CHAR}_{r,VHI} \tag{w.9b}$$

where:
$V_{r,t}^{W\_RES.lo}$ is the lower bound of volume of reservoir r at time t;
$RES_{r,VLO}^{B\_CHAR}$ is the minimum volume of reservoir r;
$V_{r,t}^{W\_RES.up}$ is the upper bound of volume of reservoir r at time t; and
$RES_{r,VHI}^{B\_CHAR}$ is the maximum volume of reservoir r.

Maximum groundwater level constraints are included to constrain aquifer levels according to physical limits:

$$GW_{g,t}^{W\_D.lo} \leq AQ_{g,MXH}^{B\_CHAR} \tag{w.10}$$

where:
$GW_{g,t}^{W\_D.lo}$ is the water table depth from ground to surface of groundwater aquifer g at time t; and
$AQ_{g,MXH}^{B\_CHAR}$ is the maximum head of groundwater aquifer g.

Finally, the reservoir volume in the last period must equal the volume selected in period one:

$$V_{r,T}^{W\_RES} = RES_{r,IVL}^{B\_CHAR} \tag{w.11}$$

5.2 Energy module

5.2.1 A detailed scheme of energy generation and distribution interlinkages

Energy generation based on different technologies and distribution of this energy among different sectors are shown in Figure 12.

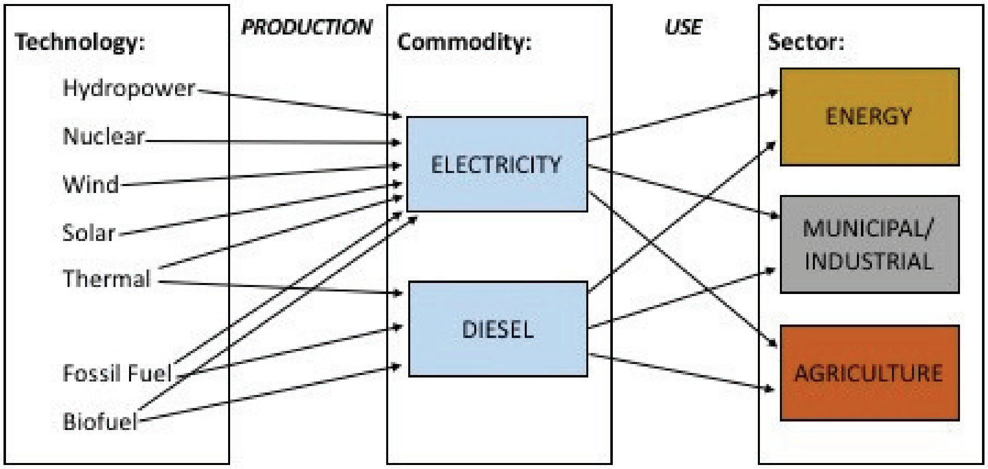

Figure 12: Energy generation and distribution system

5.2.2 Link to WSM core

The water balance at energy sites consists of inflows that come from surface and groundwater withdrawals. Some of that water is consumed or lost to evaporation, while the remainder flows back to the downstream system as drainage water, or returns to groundwater via seepage. The detailed water balance at an energy production site is depicted in Figure 13.

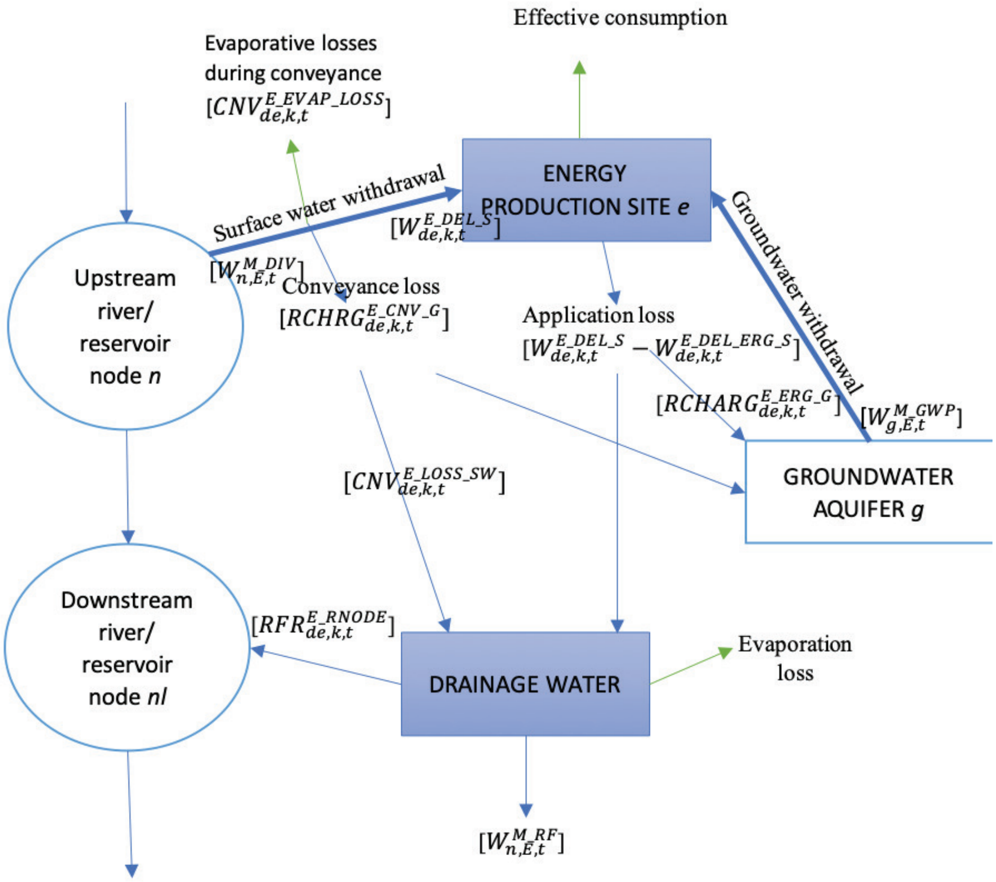

Figure 13: Water balance in an illustrative energy production site

Total surface water abstracted for energy at each site and technology depends on the surface water available:

$$W_{n,E,t}^{M\_DIV} = \sum_{de \in NDELINK} \sum_{k} \sum_{o \in KOLINK} W_{de,k,o,t}^{E\_ERG\_S} \tag{e.1}$$

where:
$W_{n,E,t}^{M\_DIV}$ is the surface water abstracted from node n at time t; and
$W_{de,k,o,t}^{E\_ERG\_S}$ is the surface water available at energy production site de (given a link $(de, n) \in NDELINK$) for technology k, to produce energy commodity o, (given a link $(o, k) \in KOLINK$) at time t.

Similarly, total groundwater abstracted for energy at each site depends on the groundwater available at each energy site and technology:

$$W_{g,E,t}^{M\_GWP} = \sum_{de \in GDELINK} \sum_{k} \sum_{o \in KOLINK} W_{de,k,o,t}^{E\_ERG\_G} \tag{e.2}$$

where:
$W_{g,E,t}^{M\_GWP}$ is the groundwater abstracted from aquifer g at time t for the energy sector; and
$W_{de,k,o,t}^{E\_ERG\_G}$ is the groundwater available at each energy production site de (given a link $(de, g) \in GDELINK$) for technology k, to produce energy commodity o, (given a link $(o, k) \in KOLINK$) at

time t.

5.2.3 Conveyance losses

Conveyance water lost to groundwater depends on the total water withdrawn and the conveyance efficiency, including efficiency gains:

$$RCHRG^{E\_CNV\_G}_{de,k,o,t} = W^{E\_ERG\_S}_{de,k,o,t} \cdot \left(1 - \left(E^{E\_CNV}_{de,k} \cdot \left(1 + \frac{E^{E\_CNV\_GN}_{de,k}}{100}\right)\right)\right) \tag{e.3}$$

where:
$RCHRG^{E\_CNV\_G}_{de,k,o,t}$ is the conveyance water lost to groundwater at energy production site de, for technology k, to produce energy commodity o, at time t;
$E^{E\_CNV}_{de,k}$ is conveyance efficiency at energy production site de for technology k; and
$E^{E\_CNV\_GN}_{de,k}$ is the gains to conveyance efficiency at energy production site de for technology k.

Conveyance water lost to evaporation further depends on evaporation:

$$CNV^{E\_EVAP\_LOSS}_{de,k,o,t} = (W^{E\_ERG\_S}_{de,k,o,t} - RCHRG^{E\_CNV\_G}_{de,k,o,t}) \cdot CNV^{E\_EVAP}_{de,k} \tag{e.4}$$

where:
$CNV^{E\_EVAP\_LOSS}_{de,k,o,t}$ is conveyance water lost to evaporation at energy production site de, for technology k, to produce energy commodity o, at time t; and
$CNV^{E\_EVAP}_{de,k}$ is evaporation at energy production site de for technology k.

Similarly, conveyance water lost to surface drainage further depends on drainage conveyance:

$$CNV^{E\_EVAP\_SW}_{de,k,o,t} = (W^{E\_ERG\_S}_{de,k,o,t} - RCHRG^{E\_CNV\_G}_{de,k,o,t} - CNV^{E\_EVAP\_LOSS}_{de,k,o,t}) \cdot CNV^{E\_DRNG}_{de,k} \tag{e.5}$$

where:
$CNV^{E\_EVAP\_SW}_{de,k,o,t}$ is conveyance water lost to surface drainage at energy production site de, for technology k, to produce energy commodity o, at time t; and
$CNV^{E\_DRNG}_{de,k}$ is drainage conveyance at energy production site de for technology k.

Finally, water returned to the river node is characterized by the fraction of water returned and the return flow:

$$RFR^{E\_RNODE}_{de,k,o,t} = RA^{E\_DIVRF}_{de,k,o} \cdot CNV^{E\_LOSS\_SW}_{de,k,o,t} \tag{e.6}$$

where:
$RFR^{E\_RNODE}_{de,k,o,t}$ is the water returned to the river node from energy production site de, using technology k, to produce energy commodity o, at time t; and
$RA^{E\_DIVRF}_{de,k,o}$ is the return flow from energy production site de, for technology k, to produce energy commodity o.

5.2.4 Total water available at energy site

The surface water delivered to an energy site to produce a given energy commodity depends on the total surface water withdrawn as well as the above outlined conveyance losses:

$$W^{E\_DEL\_S}_{de,k,o,t} = W^{E\_ERG\_S}_{de,k,o,t} - RCHRG^{E\_CNV\_G}_{de,k,o,t} - CNV^{E\_EVAP\_LOSS}_{de,k,o,t} - CNV^{E\_LOSS\_SW}_{de,k,o,t} \tag{e.7}$$

where:
$W^{E\_DEL\_S}_{de,k,o,t}$ is the total surface water delivered to energy production site de, for technology k, to produce energy commodity o, at time t.

The surface water actually available for the energy site is characterized by the application efficiency, including application efficiency gains:

$$W^{E\_DEL\_ERG\_S}_{de,k,o,t} = W^{E\_DEL\_S}_{de,k,o,t} \cdot APP^{E\_EFF}_{de,o,k}\left(1 + \frac{APP^{E\_EFF\_GN}_{de,o,k}}{100}\right) \tag{e.8}$$

where:
$W^{E\_DEL\_ERG\_S}_{de,k,o,t}$ is the surface water actually available at energy production site de, for technology k, to produce energy commodity o, at time t;
$APP^{E\_EFF}_{de,o,k}$ is the application efficiency at energy production site de, for technology k, to produce energy commodity o, at time t;
$APP^{E\_EFF\_GN}_{de,o,k}$ is the gains to application efficiency for energy production site de, for technology k, to produce energy commodity o.

Similarly the groundwater actually available to the energy site is characterized by:

$$W^{E\_DEL\_ERG\_G}_{de,k,o,t} = W^{E\_DEL\_G}_{de,k,o,t} \cdot APP^{E\_EFF}_{de,o,k}\left(1 + \frac{APP^{E\_EFF\_GN}_{de,o,k}}{100}\right) \tag{e.9}$$

where:
$W^{E\_DEL\_ERG\_G}_{de,k,o,t}$ is the groundwater actually available at energy production site de, for technology k, to produce energy commodity o, at time t; and
$W^{E\_DEL\_G}_{de,k,o,t}$ is the groundwater delivered to energy production site de, for technology k, to produce energy commodity o, at time t.

5.2.5 Total groundwater recharge

Groundwater recharge can be characterized by the total surface and groundwater delivered as well as application efficiency, including application efficiency gains:

$$RCHARGE^{E\_ERG\_G}_{de,k,o,t} = (W^{E\_DEL\_S}_{de,k,o,t} + W^{E\_DEL\_G}_{de,k,o,t}) \cdot \left(1 - \left(APP^{E\_EFF}_{de,o,k}\left(1 + \frac{APP^{E\_EFF\_GN}_{de,o,k}}{100}\right)\right)\right) \tag{e.10}$$

where:
$RCHARGE^{E\_ERG\_G}_{de,k,o,t}$ is the groundwater recharge at energy production site de, for technology k, to produce energy commodity o, at time t.

Total groundwater recharge for an energy technology and associated commodity depends on recharge from conveyance and recharge from energy site:

$$RCHARGE^{E\_TOT\_G}_{de,k,o,t} = RCHARGE^{E\_CNV\_G}_{de,k,o,t} + RCHARGE^{E\_ERG\_G}_{de,k,o,t} \tag{e.11}$$

where:
$RCHARGE^{E\_TOT\_G}_{de,k,o,t}$ is the total groundwater recharge at energy production site de, for technology k, to produce energy commodity o, at time t.

5.2.6 Return flows to WSM module

Given a link between energy production sites and nodes, $(de, n) \in NDELINK$, and a link between energy commodities and technologies $(o, k) \in KOLINK$ total return flows are characterized as:

$$W^{M\_RF}_{n,E,t} = \sum_{de \in NDELINK} \sum_{k} \sum_{o \in KOLINK} RFR^{E\_RNODE}_{de,k,o,t} \tag{e.12}$$

And total groundwater recharge from energy production sites is:

$$W^{E\_DP}_{n,E,t} = \sum_{de \in NDELINK} \sum_{k} \sum_{o \in KOLINK} RCHARG^{E\_TOT\_G}_{de,k,o,t} \tag{e.13}$$

where:
$W^{M\_RF}_{n,E,t}$ is total return flow for nodes n at time t; and $W^{E\_DP}_{n,E,t}$ is the total groundwater recharge for nodes n at time t.

5.2.7 Water demand

Water requirements at energy sites depend on requirements for energy production and the total energy produced at the site:

$$W^{E\_DEL\_ERG\_S}_{de,k,o,t} + W^{E\_DEL\_ERG\_G}_{de,k,o,t} = WATER^{E\_REQ}_{de,k,o,t} \cdot PRD^{E}_{de,k,o,t} \tag{e.14}$$

where:
$WATER^{E\_REQ}_{de,k,o,t}$: is the water required per unit of energy production at energy production site de, using technology k, to produce energy commodity o, at time t; and
$PRD^{E}_{de,k,o,t}$ is the energy produced at energy production site de, using technology k, of energy commodity type o, at time t.

5.2.8 Hydropower production

Given links between nodes and energy production sites, $(n, de) \in DENLINK$, and between nodes and reservoirs, $(n, r,) \in NRLINK$, hydropower production from reservoirs can be characterized:

$$\begin{aligned} PRD^{E}_{de,hyp,t} = & \frac{1}{1000000} \cdot 24 \cdot d^{B}_{t} \cdot G \cdot D \cdot HP^{E\_CHAR}_{de,ehpp} \sum_{n \in DENLINK} \sum_{r \in NRLINK} \\ & \left(\frac{W^{W\_TURB}_{r,t} * 1000000}{60 \cdot 60 \cdot 24 \cdot d^{B}_{t}} \cdot \left(\frac{1}{2} H^{W\_RES}_{r,t} + \frac{1}{2} H^{W\_RES}_{r,t-1} - RES^{B\_CHAR}_{r,HT0} \right) \right) \end{aligned} \tag{e.15}$$

where:
$PRD^{E}_{de,hyp,t}$ is hydropower production at energy production site de ,using reservoir systems, at time t;
d^{B}_{t} is the number of days in each month;
G is the gravitational constant $(9.81 \frac{m}{s^2})$;
D is the density of water $(998 \frac{kg}{m^3})$;
$HP^{E\_CHAR}_{de,ehpp}$ is the production efficiency of the reservoir hydropower generation facility at energy production site de;
$W^{W\_TURB}_{r,t}$ is river flow through the turbines in reservoir r at time t;
$H^{W\_RES}_{r,t}$ is the water level in reservoir r at time t; and
$RES^{B\_CHAR}_{r,HT0}$ is the tail water level for turbine discharge of reservoir r.

Similarly, hydropower production from rrun-of-the-river systems is charcterized by:

$$\begin{aligned} PRD^{E}_{de,ror,t} = & \frac{1}{1000000} \cdot 24 \cdot d^{B}_{t} \cdot HP^{E\_CHAR}_{de,eror} \cdot HP^{E\_CHAR}_{de,grhp} \\ & \cdot \sum_{n \in DENLINK} \sum_{nd} \left(\frac{W^{W\_TURB\_ROR}_{de,t} * 1000000}{60 \cdot 60 \cdot 24 \cdot d^{B}_{t}} \right) \end{aligned} \tag{e.16}$$

where:
$PRD^{E}_{de,ror,t}$ is hydropower production at energy production site de ,using run-of-the-river systems, at time t;
$HP^{E\_CHAR}_{de,eror}$ is the production efficiency of the run-of-the-river hydropower generation facility at energy production site de;
$HP^{E\_CHAR}_{de,grhp}$ is the electricity generated per unit of water at energy production site de; and
$W^{W\_TURB\_ROR}_{de,t}$ is river flow through the turbines of the run-of-the-river hydropower generation facility at energy production site de at time t.

5.2.9 Biofuel usage

Given links between nodes and energy production sites, $(n, de) \in DENLINK$, and between agricultural production sites and notes, $(da, n) \in NDALINK$, energy production from biofuels is characterized:

$$\sum_{t} PRD^{E}_{e,biof,t} = \sum_{bcr} \sum_{n \in DENLINK} \sum_{da \in NDALINK} BIO^{A\_YLD}_{da,bcr} \cdot E^{A\_BIO}_{da,bcr} \tag{e.17}$$

where:
$PRD^{E}_{e,biof,t}$ is the energy production at energy production site *de* from biofuels at time *t*;
$BIO^{A\_YLD}_{da,bcr}$ is the yield of biofuel crops (*bcr*) from agricultural production site *da*; and
$E^{A\_BIO}_{da,bcr}$ is the biofuel crop production at agricultural production site *da*.

5.2.10 Energy usage

Energy usage for water supply to energy sties depends on surface water availability and use and groundwater availability and use:

$$\begin{aligned} ENERGY^{E\_USE}_{de,o,k,o,t} =& WTR^{E\_CHAR}_{de,o,k,o,SWER} \cdot WTR^{E\_CHAR}_{de,o,k,o,SWEF} \cdot W^{E\_ERG\_S}_{de,o,k,o,t} + \\ & \sum_{g \in GDELINK} L^{E\_GPMP}_{g,o,k,o,t} \cdot WTR^{E\_CHAR}_{de,o,k,o,GWEF} \cdot W^{E\_ERG\_G}_{de,o,k,o,t} \end{aligned} \tag{e.18}$$

where:
$ENERGY^{E\_USE}_{de,o,k,o,t}$ is the energy usage for water supply to energy production site *de*, using energy commodity *o*, for technology *k*, to produce energy commodity *o*, at time *t*;
$WTR^{E\_CHAR}_{de,o,k,o,SWER}$ is the energy requirement per unit of surface water supply at energy production site *de*, using energy commodity *o*, for technology *k*, to produce energy commodity *o*;
$WTR^{E\_CHAR}_{de,o,k,o,SWEF}$ is the fraction of surface water pumped at energy production site *de*, using energy commodity *o*, for technology *k*, to produce energy commodity *o*;
$L^{E\_GPMP}_{g,o,k,o,t}$ is the energy requirement per unit of groundwater at groundwater aquifer *g*, using energy commodity *o*, for technology *k*, to produce energy commodity *o*, at time *t* (given link $(de, g) \in GDELINK$); and
$WTR^{E\_CHAR}_{de,o,k,o,GWEF}$ is the fraction of groundwater pumped at energy production site *de*, using energy commodity *o*, for technology *k*, to produce energy commodity *o*.

Total energy use in the sector depends on energy use at each site (given links $(de, n) \in NDELINK$), $(k, o) \in OKLINK$, and $(o, k) \in KOLINK$):

$$E^{M\_DIV}_{n,E,k,o,t} = \sum_{de \in NDELINK} \sum_{k \in OKLINK} \sum_{o \in KOLINK} ENERGY^{E\_USE}_{de,o,k,o,t} \cdot (1 + E^{E\_LOSS}_{de,o,k,o}) \tag{e.19}$$

where:
$E^{M\_DIV}_{n,E,k,o,t}$ is the energy withdrawn at node *n*, for the energy sector *E*, from technology *k*, to produce energy commodity *o*, at time *t*; and
$E^{E\_LOSS}_{de,o,k,o}$ is the energy lost at energy production site *de*, using energy commodity *o*, for technology *k*, to produce energy commodity *o*, at time *t*.

5.2.11 Energy balance

Given a link between energy markets and energy production sites, ($(de, m) \in MDELINK$), total energy produced must equal the sum of energy withdrawn for each sector and the energy trade balance:

$$\sum_{de \in MDELINK} PRD^{E}_{de,k,o,t} = \sum_{n \in MNLINK} \sum_{s} E^{E\_DIV}_{n,s,k,o,t} + TBAL^{E}_{m,k,o,t} \tag{e.20}$$

where:
$E^{E\_DIV}_{n,s,k,o,t}$ is the energy withdrawn at node n, for sector s, for technology k, used to produce energy commodity o, at time t; and
$TBAL^{E}_{m,k,o,t}$ is the energy trade balance in energy market m, for technology k, used to produce energy commodity o, at time t (given link between energy markets and nodes $(n, m) \in MNLINK$).

5.2.12 Energy production costs

Production costs depend on energy produced and the cost per unit:

$$C^{E\_PRD}_{de} = \sum_{k} \sum_{t} \sum_{o \in KOLINK} (PRD^{E}_{de,k,o,t} \cdot v^{E\_PROD}_{de,k,o,t}) \tag{e.21}$$

where:
$C^{E\_PRD}_{de}$ is the production cost at energy production site *de*; and
$v^{E\_PROD}_{de,k,o,t}$ is the cost per unit of energy production at energy production site *de*, using technology k, to produce energy commodity o, at time t.

Electricity transmission costs depend on the quantity of electricity transmitted and the distance from energy production site to market:

$$C^{E\_TRNS}_{de,t} = \sum_{m \in DEMLINK} pt^{E\_TRANS}_{de} \cdot et^{E\_TRANS}_{de,m} \cdot DIST^{E}_{de,m,t} \tag{e.22}$$

where:
$C^{E\_TRNS}_{de,t}$ is the transmission cost of electricity produced at energy production site *de* at time t;
$pt^{E\_TRANS}_{de,m}$ is the price of electricity transmission (in $\frac{Mwh}{m}$) from energy production site *de*;
$et^{E\_TRANS}_{de,m}$ is the distance (in m) from energy production site *de* to energy market m; and
$DIST^{E}_{de,m,t}$ is the electricity transmitted from energy production site *de*, to energy market m, (in Mwh) at time t.

Water supply costs depend on costs of surface and groundwater pumping, capacity expansion, and other costs:

$$\begin{aligned} C^{E\_WTR\_SUP}_{de} = & \sum_{k \in OKLINK} \sum_{t} \sum_{o \in KOLINK} (WTR^{E\_CHAR}_{de,k,o,SWGR} \cdot (1 - WTR^{E\_CHAR}_{de,o,k,o,SWEF}) \cdot W^{E\_ERG\_S}_{de,k,o,t} \\ & + P^{E}_{de,k,o,t} \cdot WTR^{E\_CHAR}_{de,o,k,o,SWER} \cdot WTR^{E\_CHAR}_{de,o,k,o,SWEF} + WTR^{E\_CHAR}_{de,k,o,SONC} \cdot W^{E\_ERG\_S}_{de,k,o,t} \\ & + P^{E}_{de,k,o,t}(\sum_{g \in DEGLINK} L^{E\_GPMP}_{g,o,k,o,t}) \cdot WTR^{E\_CHAR}_{de,o,k,o,GWEF} \\ & + WTR^{E\_CHAR}_{de,k,o,GONC} \cdot W^{E\_ERG\_G}_{de,k,o,t}) + C^{E\_PMXP\_S}_{de} + C^{E\_PMXP\_G}_{de} \end{aligned} \tag{e.23}$$

where: $C^{E_WTR_SUP}_{de}$is water supply cost at energy production site *de*;
$WTR^{E\_CHAR}_{de,k,o,SWGR}$ is the fixed cost of water delivery by gravity to at energy production site *de*, for technology k, to produce energy commodity o;
$WTR^{E\_CHAR}_{de,k,o,SONC}$ is other non-energy costs of conveying surface water at energy production site *de*, for technology k, to produce energy commodity o;
$P^{E}_{de,k,o,t}$ is the energy price at energy production site *de*, for energy commodity o, produced using

technology k, at time t;
$WTR^{E\_CHAR}_{de,k,o,GONC}$ is other non-energy costs of conveying groundwater at energy production site de, for technology k, to produce energy commodity o;
$C^{E\_PMXP\_S}_{de}$ is the cost of expanding surface water pumping at energy production site de; and
$C^{E_PMXP_G}_{de}$is the cost of expanding groundwater pumping at energy production site de.

The cost of expanding surface pumping is calculated:

$$C^{E\_PMXP\_S}_{de} = \sum_{k} \sum_{o \in KOLINK} WTR^{E\_CHAR}_{de,k,o,SPAC} \cdot (WTR^{E\_CHAR}_{de,k,o,SPGC})^{WTR^{E\_CHAR}_{de,k,o,SPBC}} \tag{e.24}$$

where:
$WTR^{E\_CHAR}_{de,k,o,SPAC}$ and $WTR^{E\_CHAR}_{de,k,o,SPBC}$ are parameters of surface water pumping expansion at energy production site de, for technology k, to produce energy commodity o; and
$WTR^{E\_CHAR}_{de,k,o,SPGC}$ is surface water pumping capacity growth at energy production site de, for technology k, to produce energy commodity o.

Similarly, the cost of expanding groundwater pumping is calculated:

$$C^{E\_PMXP\_G}_{de} = \sum_{k} \sum_{o \in KOLINK} WTR^{E\_CHAR}_{de,k,o,GPAC} \cdot (WTR^{E\_CHAR}_{de,k,o,GPGC})^{WTR^{E\_CHAR}_{de,k,o,GPBC}} \tag{e.25}$$

where:
$WTR^{E\_CHAR}_{de,k,o,GPAC}$ and $WTR^{E\_CHAR}_{de,k,o,GPBC}$ are parameters of groundwater pumping expansion at energy production site de, for technology k, to produce energy commodity o; and
$WTR^{E\_CHAR}_{de,k,o,GPGC}$ is groundwater pumping capacity growth at energy production site de, for technology k, to produce energy commodity o.

5.2.13 Application and conveyance efficiency

The cost of improving water application efficiency depends on the cost of technology adoption and the quantity of water saved:

$$\begin{aligned} C^{E\_APP\_EFF}_{de} = & \sum_{k} \sum_{o \in KOLINK} \Bigg(V^{E\_IREF}_{de,k,o} \Bigg(\sum_{t} \Big(W^{E\_DEL\_ERG\_S}_{de,k,o,t} + W^{E\_DEL\_ERG\_G}_{de,k,o,t} \Big) \Bigg) \\ & \cdot APP^{E\_EFF}_{de,k,o} \cdot \frac{APP^{E\_EFF\_GN}_{de,k,o}}{100} \Bigg) \end{aligned} \tag{e.26}$$

where:
$C^{E\_APP\_EFF}_{de}$ is the cost of improving water application efficiency at energy production site de; and
$V^{E\_IREF}_{de,k,o}$ is the cost of technology adoption per unit of water at energy production site de, for technology k, used to produce energy commodity o.

The costs of expanding production capacity ($C^{E\_EXPK}_{de}$) are:

$$C^{E\_EXPK}_{de} = \sum_{k} \sum_{o \in KOLINK} \alpha^{E\_EXP}_{de,k,o} (PROD^{E\_POT\_EXP}_{de,k,o})^{\beta de,k,o^{E\_EXP}} \tag{e.27}$$

where:
$\alpha^{E\_EXP}_{de,k,o}$ and $\beta de, k, o^{E\_EXP}$ are the parameters of the power production capacity expansion function at energy production site de, for technology k, used to produce energy commodity o; and $PROD^{E\_POT\_EXP}_{de,k,o}$ is the expansion gain at energy production site de, for technology k, used to produce energy commodity o.

5.2.14 Net benefits

Net benefits of energy production ($B^{M\_PRD}_{n,E}$) are calculated:

$$\begin{aligned} B^{M\_PRD}_{n,E} = \sum_{de \in NDELINK} \Bigg(& \sum_{k} \sum_{t} \sum_{o \in KOLINK} \left(P^{E}_{de,k,o,t} \cdot DIST^{E}_{de,m,t} + P^{E}_{de,agr} \cdot E^{E\_DIV\_A}_{n,k,o,t} \right) \\ & - C^{E\_PRD}_{de} - C^{E\_TRNS}_{de} - C^{E\_WTR\_SUP}_{de} - C^{E\_PMXP\_S}_{de} - C^{E\_PMXP\_G}_{de} - C^{E\_APP\_EFF}_{de} \\ & - C^{E\_CNV\_EFF}_{de} - C^{E\_EXPK}_{de} \Bigg) \end{aligned} \tag{e.28}$$

where:
$P^{E}_{de,agr}$ is the price of energy used in agriculture from energy production site de; and
$E^{E\_DIV\_A}_{n,k,o,t}$ is the energy diverted for agriculture at node n, of energy commodity o, produced by technology k, at time t.

5.2.15 Constraints

Water through turbine ($W^{E\_TURB}_{r,t}$) from reservoir r at time t cannot be more than water flowing downstream:

$$\sum_{r \in NRLINK} W^{E\_TURB}_{r,t} \leq \sum_{nd} W^{W\_F}_{n,nd,t} \tag{e.29}$$

Water through run-of-the-river turbine ($W^{E\_TURB\_ROR}_{de,t}$) at energy production site de at time t cannot be more than water flowing downstream:

$$\sum_{de \in NDELINK} W^{E\_TURB\_ROR}_{de,t} \leq \sum_{nd} W^{W\_F}_{n,nd,t} \tag{e.30}$$

Energy production cannot be greater than the capacity:

$$PRD^{E}_{de,k,o,t} \leq 24 \cdot d^{B}_{t} \cdot (PROD^{E\_POT}_{de,k,o} + PROD^{E\_POT\_EXP}_{de,k,o}) \tag{e.31}$$

Energy distribution cannot be greater than production:

$$\sum_{de} DIST^{E}_{de,m,t} \leq egreq^{UB}_{m,t} + egpop^{UB}_{m,t} \tag{e.32}$$

where:
$egreq^{UB}_{m,t}$ is the upper bound of per capita energy demand at market m and time t; and
$egpop^{UB}_{m,t}$ is the upper bound of the population getting electricity from market m at time t.

Finally, the following conditions should be fulfilled since water pumping is considered to occur either using electricity or diesel pumps:

$$\sum_{o} f^{E\_SP}_{de,o,k,o,t} = 1 \tag{e.33}$$

and

$$\sum_{o} f^{E\_GP}_{de,o,k,o,t} = 1 \tag{e.34}$$

where:
$f^{E\_SP}_{de,o,k,o,t}$ is the fraction of surface water pumped using electricity or diesel (o) for producing energy commodity o;
$f^{E\_GP}_{de,o,k,o,t}$ is the fraction of groundwater pumped using electricity or diesel (o) for producing energy commodity o.

5.3 Industry and municipality module

5.3.1 Water balance at industrial and municipal sites

The detailed water balance for an illustrative industrial production site is depicted in Figure 14. Similar to the energy module, municipal/industrial sites can draw water from groundwater and surface water sources. Some of that water is lost to evaporation and some is consumed in production or consumption processes. The remaining water returns through drainage to the surface water system or to groundwater through recharge. The water balance is presented in the equations that follow.

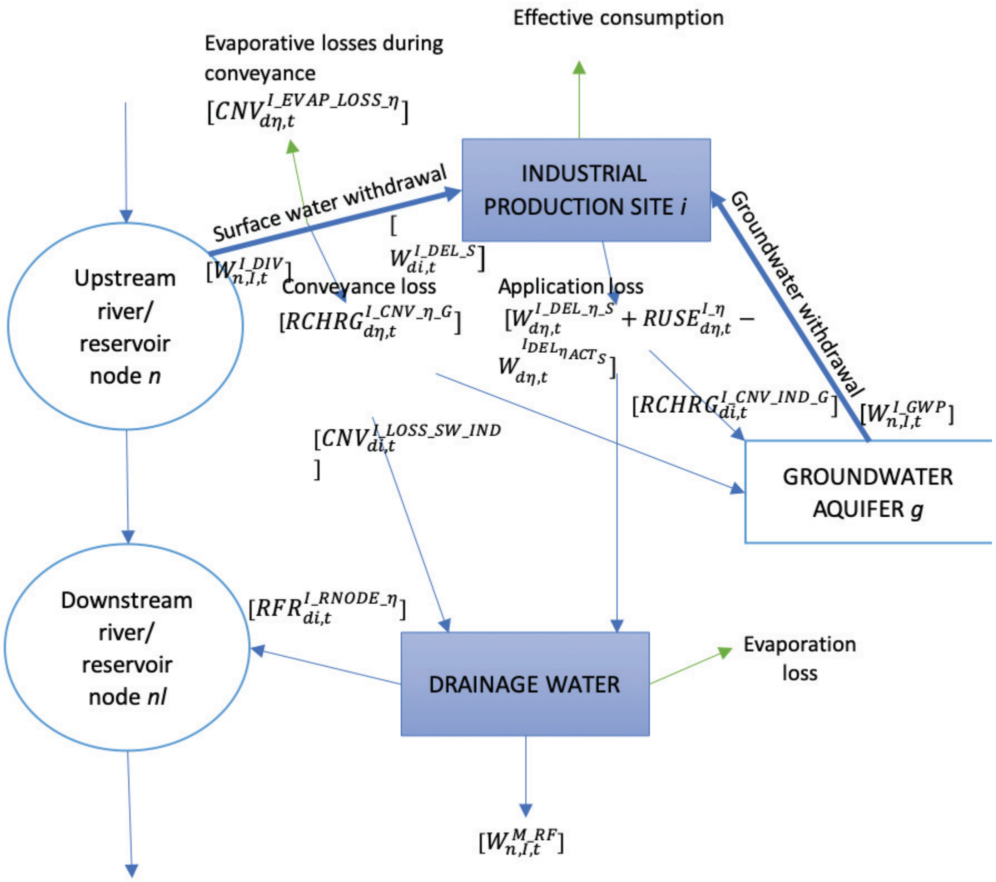

Figure 14: Water balance in an illustrative industrial production site

5.3.2 Linking to WSM module

The surface water abstracted for industrial and municipal use must be equal to the sum of the surface water available at each industry site and the surface water available at each municipal site:

$$W_{n,I,t}^{I\_DIV} = \sum_{di \in NDILINK} W_{di,t}^{I\_IND\_S} + \sum_{dm \in NDMLINK} W_{dm,t}^{I\_MUN\_S} \tag{i.1}$$

where:
$W_{n,I,t}^{I\_DIV}$ is the surface water abstracted for industrial and municipal use from node n in time t;
$W_{di,t}^{I\_IND\_S}$ is the surface water abstracted at industry site di in time t (given the link between industrial production sites and nodes $(di, n) \in NDILINK$); and
$W_{di,t}^{I\_MUN\_S}$ is the surface water abstracted at industry site dm in time t (given the link between industrial production sites and nodes $(dm, n) \in NDMLINK$).

Similarly, groundwater abstracted for industrial and municipal use must be equal to the sum of the groundwater available at each industry site and the groundwater available at each municipal site:

$$W_{n,I,t}^{I\_GWP} = \sum_{di \in NDILINK} W_{di,t}^{I\_IND\_G} + \sum_{dm \in NDMLINK} W_{dm,t}^{I\_MUN\_G} \tag{i.2}$$

where:
$W^{I\_GWP}_{n,I,t}$ is the groundwater abstracted for industrial and municipal use from node n in time t;
$W^{I\_IND\_G}_{di,t}$ is the groundwater abstracted at industry site di in time t; and
$W^{I\_MUN\_G}_{dm,t}$ is the groundwater abstracted at industry site dm in time t.

5.3.3 Conveyance losses

We consider the following characterization of conveyance losses for both the industrial and municipal sectors. For notational simplicity, we let $IND, MUN \in \eta$ to allow for these calculations in each sector. Conveyance water lost to groundwater for industrial sites depends on total surface water withdrawn and conveyance efficiency, including efficiency gains:

$$RCHRG^{I\_CNV\_\eta\_G}_{d\eta,t} = W^{W\_\eta\_S}_{d\eta,t} \cdot \left(1 - E^{I\_CNV\_\eta}_{d\eta} \cdot \left(1 + \frac{E^{I\_CNV\_\eta\_GN}_{d\eta}}{100}\right)\right) \tag{i.3}$$

where:
$RCHRG^{I\_CNV\_\eta\_G}_{d\eta,t}$ is the conveyance water lost to groundwater at industry site or municipality $d\eta$ in time g;
$E^{I\_CNV\_\eta}_{d\eta}$ is the conveyance efficiency at industry site or municipality $d\eta$; and
$E^{I\_CNV\_\eta\_GN}_{d\eta}$ is the conveyance efficiency improvement (in percentage) at industry site or municipality $d\eta$.

Conveyance water lost to evaporation depends on the total water withdraw, the water lost to groundwater, and the evaporation fraction:

$$CNV^{I\_EVAP\_LOSS\_\eta}_{d\eta,t} = (W^{W\_\eta\_S}_{d\eta,t} - RCHRG^{I\_CNV\_\eta\_G}_{d\eta,t}) \cdot CNV^{I\_EVAP\_\eta}_{d\eta} \tag{i.4}$$

where:
$CNV^{I\_EVAP\_LOSS\_\eta}_{d\eta,t}$ is the conveyance water lost to evaporation at industry site or municipality $d\eta$ and time t; and
$CNV^{I_EVAP_\eta}_{d\eta}$i s the conveyance evaporation loss fraction at industry site or municipality $d\eta$.

Total conveyance water lost to surface drainage at an industrial site or municipality depends on groundwater and evaporation loss as well as the fraction of water lost to surface drainage:

$$CNV^{I\_LOSS\_SW\_\eta}_{d\eta,t} = (W^{W\_\eta\_S}_{d\eta,t} - RCHRG^{I\_CNV\_\eta\_G}_{d\eta,t} - CNV^{I\_EVAP\_LOSS\_\eta}_{d\eta,t}) \cdot CNV^{I\_DRNG\_\eta}_{d\eta} \tag{i.5}$$

where:
$CNV^{I\_LOSS\_SW\_\eta}_{d\eta,t}$ is the conveyance water lost to surface drainage at industry site or municipality $d\eta$ and time t; and
$CNV^{I\_DRNG\_\eta}_{d\eta}$ is the conveyance lost to surface drainage fraction at industry site or municipality $d\eta$.

Finally, water returned to the river node from conveyance depends on the fraction of water returned and the return flow:

$$RFR_{d\eta,t}^{I\_RNODE\_\eta} = RA_{d\eta}^{I\_DIVRF\_\eta} \cdot CNV_{d\eta}^{I\_LOSS\_SW\_\eta} \tag{i.6}$$

where:
$RFR_{d\eta,t}^{I\_RNODE\_\eta}$ is water returned to the river node from conveyance at industry site or municipality $d\eta$ in time t; and
$RA_{d\eta}^{I\_DIVRF\_\eta}$ is the fraction of return flow returned to the river node at industry site or municipality $d\eta$.

5.3.4 Water reuse after wastewater treatment

Surface water delivered for industry site or municipality $d\eta$ depends on the total surface water withdrawn and conveyance groundwater, evaporation, and surface drainage loss:

$$W_{d\eta}^{I\_DEL\_\eta\_S} = W_{d\eta}^{I\_\eta\_S} - RCHRG_{d\eta,t}^{I\_CNV\_\eta\_G} - CNV_{d\eta,t}^{I\_EVAP\_LOSS\_\eta} - CNV_{d\eta,t}^{I\_LOSS\_SW\_\eta} \tag{i.7}$$

Surface water actually available depends on the water delivered and reused as well as application efficiency, including efficiency gains:

$$W_{d\eta}^{I\_DEL\_\eta\_ACT\_S} = (W_{d\eta,t}^{I\_DEL\_\eta\_S} + RUSE_{d\eta,t}^{I\_\eta}) \cdot APP_{d\eta}^{I\_EFF\_\eta}\left(1 + \frac{APP_{d\eta}^{I\_EFF\_\eta\_GN}}{100}\right) \tag{i.8}$$

where:
$W_{d\eta}^{I\_DEL\_\eta\_ACT\_S}$ is surface water actually available at industry site or municipality $d\eta$ and time t;
$RUSE_{d\eta,t}$ is reused water at industry site or municipality $d\eta$ and time t;
$APP_{d\eta}^{I\_EFF\_\eta}$ is application efficiency at industry site or municipality $d\eta$; and
$APP_{d\eta}^{I\_EFF\_\eta\_GN}$ is application efficiency at industry site or municipality $d\eta$.

Similarly, groundwater actually available can be calculated:

$$W_{d\eta,t}^{I\_DEL\_\eta\_G} = W_{d\eta,t}^{I\_\eta\_G} \cdot APP_{d\eta}^{I\_EFF\_\eta}\left(1 + \frac{APP_{d\eta}^{I\_EFF\_\eta\_GN}}{100}\right) \tag{i.9}$$

where:
$W_{d\eta,t}^{I\_DEL\_\eta\_G}$ is groundwater actually available at industry site or municipality $d\eta$ in time t.

Return flow from industrial site or municipality after application depends on total water availability (surface, ground, and reuse) and application efficiency including efficiency gains:

$$\begin{aligned} RTN_{d\eta,t}^{I\_\eta\_S} =& (W_{d\eta,t}^{I\_DEL\_\eta\_S} + RUSE_{d\eta,t}^{I\_\eta} + W_{d\eta,t}^{I\_\eta\_G}) \\ & \cdot \left(1 - APP_{d\eta}^{I\_EFF\_\eta} \cdot \left(1 + \frac{APP_{d\eta}^{I\_EFF\_\eta\_GN}}{100}\right)\right) \end{aligned} \tag{i.10}$$

where:
$RTN_{d\eta,t}^{I\_\eta\_S}$ is the return flow from industry site or municipality $d\eta$ and time t.

5.3.5 Water reuse after wastewater treatment

Total water reused is calculated from the return flow from industrial site or municipality after application and the fraction of reuse water:

$$RUSE_{d\eta,t}^{I\_\eta} = RTN_{d\eta,t}^{I\_\eta\_S} \cdot RUSE_{d\eta,t}^{I\_FRC\_\eta} \tag{i.11}$$

where:
$RUSE_{d\eta,t}^{I\_FRC\_\eta}$ is the fraction of reuse water at industry site or municipality $d\eta$ and time t.

5.3.6 Return flow back to WSM module

The return flow from industrial or municipal site in million cubic meters $(W_{n,I,t}^{M\_RF})$ depends on the return flow from each site:

$$\begin{aligned} W_{n,I,t}^{M\_RF} = & \sum_{di \in NDILINK} (RFR_{di,t}^{I\_RNODE\_IND} + RTN_{di,t}^{I\_IND\_S}) \\ & + \sum_{dm \in NDMLINK} (RFR_{dm,t}^{I\_RNODE\_MUN} + RTN_{dm,t}^{I\_MUN\_S}) \end{aligned} \tag{i.12}$$

Similarly, total groundwater recharge $(W_{n,I,t}^{M\_DP})$ is calculated as the sum of groundwater recharge from each industrial and municipal site:

$$W_{n,I,t}^{M\_DP} = \sum_{di \in NDILINK} RCHRG_{di,t}^{I\_CNV\_IND\_G} + \sum_{dm \in NDMLINK} RCHRG_{dm,t}^{I\_CNV\_MUN\_G} \tag{i.13}$$

5.3.7 Water demand

The industrial water requirement depends on the water requirement per unit of production as well as total production:

$$WATER_{di,t}^{I\_DMD\_IND} = WATER_{di,t}^{I\_IND\_REQ} \cdot \sum_{p} IND_{di,t,p}^{I\_PROD\_POT} \tag{i.14}$$

where: $WATER_{di,t}^{I\_DMD\_IND}$ is the industrial water requirement based on potential production at site di and time t;
$WATER_{di,t}^{I\_IND\_REQ}$ is the water required per unit of industrial production at site di and time t and
$IND_{di,t,p}^{I\_PROD\_POT}$ is the potential industrial production of good p, at industrial site di, at time t.

Municipal water demand depends on the water requirement per person and the population of the municipality:

$$WATER_{dm,t}^{I\_DMD\_MUN} = WATER_{dm,t}^{I\_MUN\_REQ} \cdot POP_{dm,t}^{I} \tag{i.15}$$

where:
$WATER^{I\_DMD\_MUN}_{dm,t}$ is the municipal water requirement based on population at municipality dm and time t;
$WATER^{I\_MUN\_REQ}_{dm,t}$ is the water required per person (m^3/person) at municipality dm and time t; and
$POP^{I}_{dm,t}$ is the population of municipality dm at time t.

The reduction ratio in industrial production maintains the water constraint condition, taking into account the water demands and the sum of actual surface water and groundwater delivered. This is calculated:

$$IND^{I\_PROD\_RED\_RATIO\_WTR}_{di,t} = \frac{W^{I\_DEL\_IND\_ACT\_S}_{di,t} + W^{I\_DEL\_IND\_ACT\_G}_{di,t}}{WATER^{I\_DMD\_IND}_{di,t}} \tag{i.16}$$

Similarly, the reduction ratio in the municipal water requirement is calculated:

$$MUN^{I\_PROD\_RED\_RATIO\_WTR}_{dm,t} = \frac{W^{I\_DEL\_MUN\_ACT\_S}_{dm,t} + W^{I\_DEL\_MUN\_ACT\_G}_{dm,t}}{WATER^{I\_DMD\_MUN}_{dm,t}} \tag{i.17}$$

where:
$IND^{I\_PROD\_RED\_RATIO\_WTR}_{di,t}$ is the reduction ratio in industrial production at industrial site di at time t; and
$MUN^{I\_PROD\_RED\_RATIO\_WTR}_{dm,t}$ is the reduction ratio in the municipal requirement at municipality dm at time t.

5.3.8 Energy usage

Energy usage for water supply to industrial sites depends on the energy requirements for each type of water (surface, ground, reuse, and waste) the fraction of water pumped or treated, and the amount of each type of water used:

$$\begin{aligned} ENERGY^{I\_ENG\_CHAR\_\eta}_{d\eta,k,o,t} =& WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWER} \cdot WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWEF} \cdot W^{I\_\eta\_S}_{d\eta,t} \\ &+ \left(\sum_{g \in D\eta GLINK} L^{E\_GPMP}_{g,k,o,t} \right) \cdot WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GWEF} \cdot W^{I\_\eta\_G}_{d\eta,t} \\ &+ WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,WUER} \cdot RUSE^{I\_\eta}_{d\eta,t} \\ &+ WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,WWTR} \cdot WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,WWFR} \cdot RTN^{I\_\eta\_S}_{d\eta,t} \end{aligned} \tag{i.18}$$

where:
$ENERGY^{I\_ENG\_CHAR\_\eta}_{d\eta,k,o,t}$ is the energy usage for water supply at industry site or municipality $d\eta$, using energy commodity o, produced by technology k, at time t;
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWER}$ is the energy required to deliver a unit of surface water at industry site or municipality $d\eta$, using energy commodity o, produced by technology k;
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWEF}$ is the fraction of surface water pumped at industry site or municipality $d\eta$,using energy commodity o, produced by technology k;

$L^{E\_GPMP}_{g,k,o,t}$ is the energy required to pump on unite of groundwater at site g, using energy commodity o, produced by technology k, at time t (given the link between groundwater aquifers and industrial production sites or municipalities $(g, d\eta) \in D\eta GLINK$);
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GWEF}$ is the fraction of groundwater pumped at industry site or municipality $d\eta$, using energy commodity o, produced by technology k;
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,WUER}$ is the energy required to deliver a unit of reuse water to industry site or municipality $d\eta$ using energy commodity o, produced by technology k;
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,WWTR}$ is the energy required to deliver a unit of waste water at industry site or municipality $d\eta$ using energy commodity o, produced by technology k;
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,WWFR}$ is the fraction of waste water pumped industry site or municipality $d\eta$ using energy commodity o, produced by technology k.

Energy usage at industrial sites depends on the energy required per unit of industrial production as well as total production:

$$ENERGY^{I\_USE\_PROD\_IND}_{di,k,o,t} = ENERGY^{I\_IND\_REQ}_{di,k,o,t} \cdot \sum_{p} ACT^{I\_IND\_PROD}_{di,t,p} \tag{i.19}$$

where:
$ENERGY^{I\_USE\_PROD\_IND}_{di,k,o,t}$ is non-water energy usage at industrial site di, using energy commodity o, produced by technology k, at time t;
$ENERGY^{I\_IND\_REQ}_{di,k,o,t}$ is energy required per unit of industrial production at industrial site di, using energy commodity o, produced by technology k, at time t;
$ACT^{I\_IND\_PROD}_{di,t,p}$ is actual production of good p, at industrial site di, at time t.

And energy usage for municipalities depends on the energy requirement per capita and the population supported:

$$ENERGY^{I\_USE\_PROD\_MUN}_{dm,k,o,t} = ENERGY^{I\_MUN\_REQ}_{dm,k,o,t} \cdot ACT^{I\_POP\_WITH\_ERG}_{dm,t} \tag{i.20}$$

where:
$ENERGY^{I\_USE\_PROD\_MUN}_{dm,k,o,t}$ is non-water energy usage for municipality dm, using energy commodity o, produced by technology k, at time t;
$ENERGY^{I\_MUN\_REQ}_{dm,k,o,t}$ is energy required per capita for municipality dm, using energy commodity o, produced by technology k, at time t;
$ACT^{I\_POP\_WITH\_ERG}_{dm,t}$ is actual population supported at municipality dm at time t.

5.3.9 Energy demand

Industrial energy requirement is calculated as:

$$ERG^{I\_DMD\_IND}_{di,k,o,t} = ENERGY^{I\_IND\_REQ}_{di,k,o,t} \cdot f^{I\_PROD\_POT}_{di,t,p}(x_{di,t,p}, y_{di,t,p}) \tag{i.21}$$

where:
$ERG^{I\_DMD\_IND}_{di,k,o,t}$ is the industrial energy requirement at industrial site di, using energy commodity o, produced by technology k, at time t; and

$f^{I\_PROD\_POT}_{di,t,p}(x_{di,t,p}, y_{di,t,p})$ is the production function for industrial production for good p, at industrial site di, at time t, and $x_{di,t,p}$ and $y_{di,t,p}$ are the factors of production.

The municipal energy requirement is calculated similarly:

$$ERG^{I\_DMD\_MUN}_{dm,k,o,t} = ENERGY^{I\_MUN\_REQ}_{dm,k,o,t} \cdot IND^{I\_POP}_{dm,t} \tag{i.22}$$

where:
$ERG^{I\_DMD\_MUN}_{dm,k,o,t}$ is the municipal energy requirement for municipality dm, using energy commodity o, produced by technology k, at time t; and
$IND^{I\_POP}_{dm,t}$ is the total population of municipality dm at time t.

The reduction ratio in industrial production maintains the energy constraint condition, taking into account the energy demands and the sum of energy usage for water supply and non-water energy usage at the industrial site. This is calculated:

$$\begin{aligned} &IND^{I\_PROD\_RED\_RATIO\_ERG}_{di,t} = \\ &\sum_{k} \sum_{o \in KOLINK} \frac{ENERGY^{I\_USE\_WTR\_IND}_{di,k,o,t} + ENERGY^{I\_USE\_PROD\_IND}_{di,k,o,t}}{ERG^{I\_DMD\_IND}_{di,k,o,t}} \end{aligned} \tag{i.23}$$

Similarly, the reduction ratio in the municipal water requirement is calculated:

$$\begin{aligned} &MUN^{I\_PROD\_RED\_RATIO\_ERG}_{dm,t} = \\ &\sum_{k} \sum_{o \in KOLINK} \frac{ENERGY^{I\_USE\_WTR\_MUN}_{dm,k,o,t} + ENERGY^{I\_USE\_PROD\_MUN}_{dm,k,o,t}}{ERG^{I\_DMD\_MUN}_{dm,k,o,t}} \end{aligned} \tag{i.24}$$

where:
$IND^{I\_PROD\_RED\_RATIO\_ERG}_{di,t}$ is the reduction ration in industrial production at industrial site di at time t; and
$MUN^{I\_PROD\_RED\_RATIO\_ERG}_{dm,t}$ is the reduction ratio for municipality dm at time t.

5.3.10 Energy balance

Given the links between industrial production sties and nodes $(di, n) \in NDILINK$ and between municipal sites and nodes $(dm, n) \in NDMLINK$, total energy demand in the industrial and municipal sectors depends on the energy use and energy loss at each industrial or municipal site:

$$\begin{aligned} E^{M\_DIV}_{n,I,k,o,t} = &\sum_{di \in NDILINK} \big((ENERGY^{I\_USE\_WTR\_IND}_{di,k,o,t} + ENERGY^{I\_USE\_PROD\_IND}_{di,k,o,t}) \\ &\cdot (1 + E^{I\_LOSS\_IND}_{di,k,o})\big) + \sum_{dm \in NDMLINK} \big((ENERGY^{I\_USE\_WTR\_MUN}_{dm,k,o,t} \\ &+ ENERGY^{I\_USE\_PROD\_MUN}_{dm,k,o,t}) \cdot (1 + E^{I\_LOSS\_MUN}_{dm,k,o})\big) \end{aligned} \tag{i.25}$$

where:
$E^{I\_LOSS\_IND}_{di,k,o}$ is energy loss at industrial site di, using energy commodity o, produced by technology

k; and
$E^{I\_LOSS\_MUN}_{di,k,o}$ is energy loss at municipal site dm, using energy commodity o, produced by technology k.

5.3.11 Actual industry production and municipal population supported

The actual industrial production depends on the greatest production constraint (water or energy) and the potential industrial production. This is calculated for each technology k:

$$\begin{aligned} ACT^{I\_IND\_PROD}_{di,t,p} = &\min(IND^{I\_PROD\_RED\_RATIO\_WTR}_{di,t}, IND^{I\_PROD\_RED\_RATIO\_ERG}_{di,t}) \\ &\cdot f^{I\_PROD\_POT}_{di,t,p}(x_{di,t,p}, y_{di,t,p}) \end{aligned} \tag{i.26}$$

The actual population with access to energy from energy commodity o is calculated:

$$ACT^{I\_POP\_WITH\_ERG}_{dm,t} = MUN^{I\_RED\_RATIO\_ERG}_{dm,t} \cdot POP^{I}_{dm,t} \tag{i.27}$$

5.3.12 Industry and municipality production costs

Water supply costs depend on the fixed cost of water delivery by gravity, the energy costs of surface water conveyance and groundwater pumping, the costs of expanding pumping capacity, and other costs:

$$\begin{aligned} C^{I\_WTR\_SUP\_\eta}_{d\eta} = &\sum_t \Bigg(FXD^{I\_C\_WTR\_GRAVITY\_\eta}_{d\eta} \cdot \left(1 - \sum_o WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWEF}\right) \cdot W^{I\_\eta\_S}_{d\eta,t} \\ &+ \sum_k \sum_{o \in KOLINK} \Bigg(\Bigg(\sum_{e \in D\eta DELNK} P^{E}_{de,k,o,t} \Bigg) \cdot WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWER} \\ &\cdot WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SWEF} + WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SONC} \Bigg) \cdot W^{I\_\eta\_S}_{d\eta,t} \\ &\sum_k \sum_{o \in KOLINK} \Bigg(\Bigg(\sum_{e \in D\eta DELINK} P^{E}_{de,k,o,t} \Bigg) \cdot \Bigg(\sum_{g \in D\eta GLINK} L^{E}_{g,o,t} \Bigg) \\ &\cdot WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GWEF} + WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GONC} \Bigg) \cdot W^{I\_\eta\_G}_{d\eta,t} \Bigg) + C^{I\_PMXP\_\eta\_S}_{d\eta} \\ &+ C^{I\_PMXP\_\eta\_G}_{d\eta} \end{aligned} \tag{i.28}$$

where:
$C^{I\_WTR\_SUP\_\eta}_{d\eta}$ is the water supply cost at industry site or municipality $d\eta$;
$FXD^{I\_C\_WTR\_GRAVITY\_\eta}_{d\eta}$ is the fixed cost of water delivery by gravity at industry site or municipality $d\eta$;
$P^{E}_{de,k,o,t}$ is the energy price at site de, for energy commodity o, produced using technology k, at time t (given the link between industry site or municipality $d\eta$ and energy production sites $(de, d\eta) \in D\eta DELINK$);
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SONC}$ is other non-energy costs of conveying surface water at industry site or municipality $d\eta$;

$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GONC}$ is other non-energy costs of conveying groundwater at industry site or municipality $d\eta$;
$C^{I\_PMXP\_\eta\_S}_{d\eta}$ is the cost of expanding surface water pumping for industry site or municipality $d\eta$; and
$C^{I\_PMXP\_\eta\_G}_{d\eta}$ is the cost of expanding groundwater pumping for industry site or municipality $d\eta$.

The cost of expanding surface water pumping is calculated:

$$C^{I\_PMXP\_\eta\_S}_{d\eta} = \sum_{k} \sum_{o \in KOLINK} WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SPAC} \cdot (WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SPGC})^{WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SPBC}} \tag{i.29}$$

where:
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SPAC}$ and $WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SPBC}$ are parameters for expansion of surface water capacity at industry site or municipality $d\eta$, for energy commodity o, produced using technology k; and
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,SPGC}$ is surface water pumping capacity growth at industry site or municipality $d\eta$, for energy commodity o, produced using technology k.

Similarly, the cost of expanding groundwater pumping is calculated:

$$C^{I\_PMXP\_\eta\_G}_{d\eta} = \sum_{k} \sum_{o \in KOLINK} WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GPAC} \cdot (WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GPGC})^{WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GPBC}} \tag{i.30}$$

where:
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GPAC}$ and $WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GPBC}$ are parameters for expansion of groundwater capacity at industry site or municipality $d\eta$, for energy commodity o, produced using technology k; and
$WTR^{I\_ERG\_CHAR\_\eta}_{d\eta,k,o,GPGC}$ is groundwater pumping capacity growth at industry site or municipality $d\eta$, for energy commodity o, produced using technology k.

The cost of improving water application efficiency depends on the cost of technology adoption and the quantity of water saved:

$$C^{I\_CNV\_EFF\_\eta}_{d\eta} = V^{I\_CNEF\_\eta}_{d\eta} \cdot \sum_{t} (W^{I\_DEL\_\eta\_S}_{d\eta,t}) \cdot E^{I\_CNV\_\eta}_{d\eta} \cdot \frac{APP^{I\_EFF\_\eta\_GN}_{d\eta}}{100} \tag{i.31}$$

where:
$C^{I\_CNV\_EFF\_\eta}_{d\eta}$ is the cost of improving water application efficiency for industry site or municipality $d\eta$; and
$V^{I\_CNEF\_\eta}_{d\eta}$ is the cost of technology adoption (per unit of water) for industry site or municipality $d\eta$.

Water treatment costs depend on the quantity of treated water. This is calculated for surface water:

$$C^{I\_WTR\_TREAT\_\eta\_S}_{d\eta} = \sum_{t} (V^{I\_TRT\_\eta\_S}_{d\eta} \cdot W^{I\_\eta\_S}_{n\eta,t}) \tag{i.32}$$

and for groundwater:

$$C^{I\_WTR\_TREAT\_\eta\_G}_{d\eta} = \sum_{t} (V^{I\_TRT\_\eta\_G}_{d\eta} \cdot W^{I\_\eta\_G}_{n\eta,t}) \tag{i.33}$$

where:
$C_{d\eta}^{I\_WTR\_TREAT\_\eta\_S}$ is the cost of surface water treatment at industry site or municipality $d\eta$;
$V_{d\eta}^{I\_TRT\_\eta\_S}$ is the treatment cost per unit of surface water at industry site or municipality $d\eta$;
$C_{d\eta}^{I\_WTR\_TREAT\_\eta\_G}$ is the cost of groundwater treatment at industry site or municipality $d\eta$; and
$V_{d\eta}^{I\_TRT\_\eta\_G}$ is the treatment cost per unit of groundwater at industry site or municipality $d\eta$.

Wastewater treatment cost for reuse depends on the quantity of water reused:

$$C_{d\eta}^{I\_WWTR\_RUSE\_TREAT\_\eta} = \sum_t (V_{d\eta}^{I\_RUSE\_WWTR\_TRT\_\eta} \cdot RUSE_{d\eta,t}^{I\_\eta}) \tag{i.34}$$

where:
$C_{d\eta}^{I\_WWTR\_RUSE\_TREAT\_\eta}$ is the cost of wastewater treatment for reuse at industry site or municipality $d\eta$; and
$V_{d\eta}^{I\_RUSE\_WWTR\_TRT\_\eta}$ is the wastewater treatment cost per unit of water reused at industry site or municipality $d\eta$.

Similarly, wastewater treatment cost for return flow depends on the treatment costs and the return flow, not counting reuse water:

$$C_{d\eta}^{I\_WWTR\_TREAT\_\eta} = \sum_t (V_{d\eta}^{I\_WWTR\_TRT\_\eta} \cdot (RTN_{d\eta,t}^{I\_\eta\_S} - RUSE_{d\eta,t}^{I\_\eta})) \tag{i.35}$$

where:
$C_{d\eta}^{I\_WWTR\_TREAT\_\eta}$ is the cost of wastewater treatment at industry site or municipality $d\eta$; and
$V_{d\eta}^{I\_WWTR\_TRT\_\eta}$ is the wastewater treatment cost per unit of water at industry site or municipality $d\eta$.

Other production costs are calculated based on total production for the industrial sector:

$$C_{di}^{I\_OTR\_PROD\_IND} = \sum_t \sum_p (V_{di,p}^{I\_OTR\_PROD\_IND} \cdot ACT_{di,t,p}^{I\_IND\_PROD}) \tag{i.36}$$

where:
$C_{di}^{I\_OTR\_PROD\_IND}$ is other production cost at industrial site di; and
$V_{di,p}^{I\_OTR\_PROD\_IND}$ is other production cost per unit of production of good p, at industrial site di, at time t.

This is calculated similarly for municipalities:

$$C_{di}^{I\_OTR\_PROD\_MUN} = \sum_t (V_{dm}^{I\_OTR\_PROD\_MUN} \cdot POP_{dm,t}^{I}) \tag{i.37}$$

where:
$C_{di}^{I\_OTR\_PROD\_MUN}$ is other production cost for municipality dm; and
$POP_{dm,t}^{I}$ is the population in municipality dm at time t.

Given the above calculations, total production costs for industrial sites or municipalities $(C_{d\eta}^{I\_TOT\_\eta})$

are calculated:

$$\begin{aligned}C_{d\eta}^{I\_TOT\_\eta} =&C_{d\eta}^{I\_WTR\_SUP\_\eta} + C_{d\eta}^{I\_APP\_EFF\_\eta} + C_{d\eta}^{I\_CNV\_EFF\_\eta} + C_{d\eta}^{I\_WTR\_TREAT\_\eta\_S} \\ &+ C_{d\eta}^{I\_WTR\_TREAT\_\eta\_G} + C_{d\eta}^{I\_WWTR\_RUSE\_TREAT\_\eta} + C_{d\eta}^{I\_WWTR\_TREAT\_\eta} \\ &+ C_{d\eta}^{I\_OTR\_PROD\_\eta}\end{aligned} \tag{i.38}$$

5.3.13 Net benefits

The net benefits $(B_{n,I}^{M\_PRD})$ in this module depend on the total production value of industry as well as industrial and municipal costs:

$$\begin{aligned}B_{n,I}^{M\_PRD} =& \sum_{di\in NDILINK} \sum_{t} \sum_{p} R_{di,t,p}^{I} ACT_{di,t,p}^{I\_IND\_PROD} \\ &+ \sum_{dm\in NDMLINK} \int_{0}^{WT_{dm}^{I}} A \cdot (WP_{dm}^{I})^{\alpha} dWP_{dm}^{I} - \sum_{di\in NDILINK} C_{di}^{I\_TOT\_IND} \\ &- \sum_{dm\in NDMLINK} C_{dm}^{I\_TOT\_MUN}\end{aligned} \tag{i.39}$$

where:
$R_{di,t,p}^{I}$ is the price of good p, at industrial site di, at time t;
WT_{dm}^{I} is the water tariff for municipality dm; and
$A \cdot (WP_{dm}^{I})^{\alpha}$ is the demand curve for water for municipality dm, with A being a constant, WP_{dm}^{I} the price of water, and α the price elasticity of demand.

5.3.14 Constraints

Surface and groundwater supply can occur using electricity pumps or diesel pumps:

$$W_{d\eta,t}^{I\_\eta\_S} = \sum_{k} \sum_{o\in KOLINK} WTR_{d\eta,k,o,SWEF}^{I\_ERG\_CHAR\_\eta} \cdot W_{d\eta,t}^{I\_\eta\_S} \tag{i.40}$$

$$W_{d\eta,t}^{I\_\eta\_G} = \sum_{k} \sum_{o\in KOLINK} WTR_{d\eta,k,o,GWEF}^{I\_ERG\_CHAR\_\eta} \cdot W_{d\eta,t}^{I\_\eta\_G} \tag{i.41}$$

Water treatment can occur using electricity or diesel pumps:

$$RTN_{d\eta,t}^{I\_\eta\_S} = \sum_{k} \sum_{o\in KOLINK} TR_{d\eta,k,o,WWFR}^{I\_ERG\_CHAR\_\eta} \cdot RTN_{d\eta,t}^{I\_\eta\_S} \tag{i.42}$$

5.4 Agriculture module

5.4.1 Water balance

The water balance at irrigation nodes includes conveyance, effective consumption, deep percolation and return flow relationships (Figure 15). Surface water withdrawn for irrigation needs is partially lost during conveyance. This conveyance loss is composed of non-productive evaporation losses, seepage to groundwater aquifers, and flow to the drainage system. Crop water demand can be met using surface water, pumping of groundwater, or through reuse of drainage water. Crops also consume water from precipitation. Finally, the water balance must account for the fact that only some of the water delivered to the field level is effectively used by crops, with the remaining water being lost through deep percolation back into groundwater. Return flows (drainage waters) are also split between the river, non-productive evaporation loss, and flows into other depressions located at the ends of irrigation canals. The equations below describe this water balance.

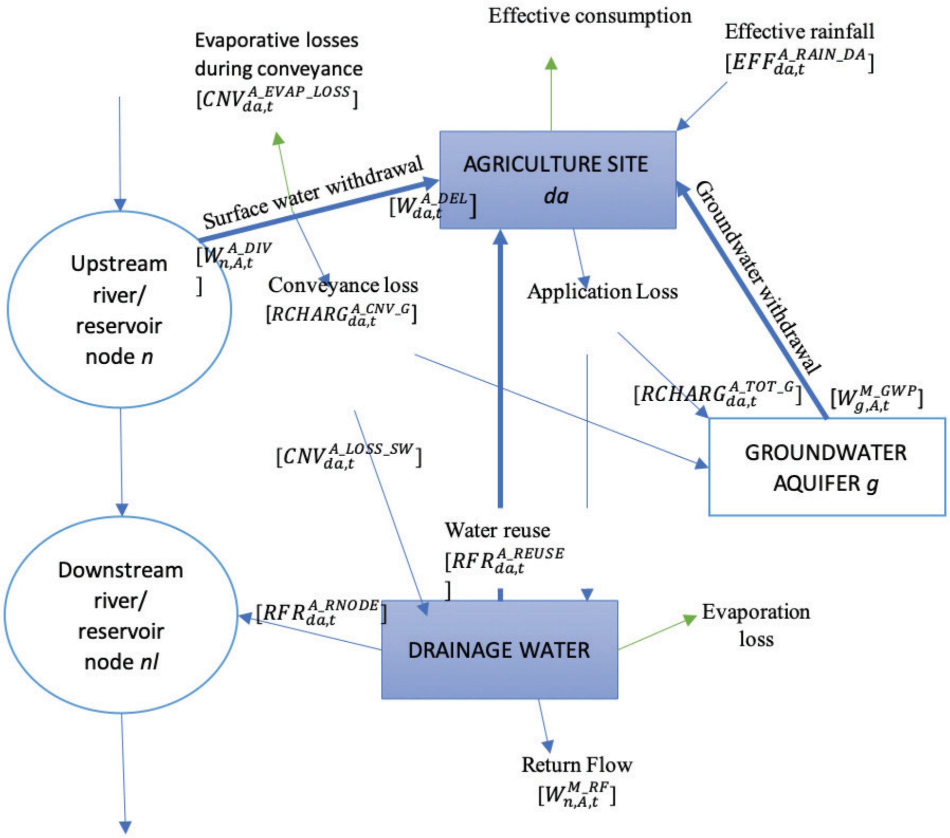

Figure 15: Water balance in an illustrative agricultural site

Total effective rainfall at a particular node is the sum of effective rainfall of all associated agricultural nodes[7]:

$$EFF_{da,t}^{A\_RAIN\_DA} = \sum_{n \in DANLINK} EFF_{n,t}^{A\_RAIN} \tag{f.1}$$

where:
$EFF_{da,t}^{A\_RAIN\_DA}$ is the total effective rainfall at agriculture production site da at time t; and
$EFF_{n,t}^{A\_RAIN}$ is the effective rainfall at node n at time t (given the link between nodes and agriculture production sites $(n, da) \in DANLINK$).

Similarly, potential evapotranspiration $(PET_{da,t}^{A\_DA})$ is calculated:

$$PET_{da,t}^{A\_DA} = \sum_{n \in DANLINK} PET_{n,t}^{W} \tag{f.2}$$

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[7] For effective rainfall calculation, see equations Af.1a-Af.1e in the Appendix.

where:
$PET^{W}_{n,t}$ is the potential evapotranspiration within the catchment at time t.

5.4.2 Linking to WSM module

Surface water abstracted for agriculture is calculated as:

$$W^{W\_DIV}_{n,A,t} = \sum_{da \in NDALINK} W^{A\_AGG\_S}_{da,t} \tag{f.3}$$

where:
$W^{W\_DIV}_{n,A,t}$ is water withdrawn for agriculture use from node n at time t; and
$W^{A\_AGG\_S}_{da,t}$ is the surface water abstracted for agriculture production site da at time t (given the link between agriculture production sites and nodes $(da, n) \in NDALINK$).

Similarly groundwater abstracted for agriculture is calculated:

$$W^{W\_GWP}_{n,A,t} = \sum_{da \in NDALINK} W^{A\_AGG\_G}_{da,t} \tag{f.4}$$

where:
$W^{W\_GWP}_{n,A,t}$ is groundwater pumping for agriculture use from groundwater aquifer g at time t; and
$W^{A\_AGG\_G}_{da,t}$ is the groundwater abstracted for agriculture production site da at time t (given the link between agriculture production sites and nodes $(da, g) \in GDALINK$).

5.4.3 Conveyance losses

Conveyance water lost to groundwater depends on the total water withdrawn and the conveyance efficiency, including efficiency gains:

$$RCHARG^{A\_CNV\_G}_{da,t} = W^{A\_AGG\_S}_{da,t} \cdot \left(1 - \left(E^{A\_CNV}_{da} \cdot \left(1 + \frac{E^{A\_CNV\_GN}_{da}}{100}\right)\right)\right) \tag{f.5}$$

where:
$RCHARG^{A\_CNV\_G}_{da,t}$ is the conveyance water lost to groundwater at agriculture production site da at time t;
$E^{A\_CNV}_{da}$ is the conveyance efficiency at agriculture production site da; and
$E^{A\_CNV\_GN}_{da}$ is the conveyance efficiency improvement over the original at agriculture production site da.

Conveyance water lost to evaporation depends on the total water withdraw, the water lost to groundwater, and the evaporation fraction:

$$CNV^{A\_EVAP\_LOSS}_{da,t} = (W^{A\_AGG\_S}_{da,t} - RCHARG^{A\_CNV\_G}_{da,t}) \cdot CNV^{A\_EVAP}_{da} \tag{f.6}$$

where:
$CNV^{A\_EVAP\_LOSS}_{da,t}$ is the conveyance water lost to evaporation at agriculture production site da at

time t; and
$CNV_{da}^{A\_EVAP}$ is the conveyance evaporation loss fraction at agriculture production site da.

Total conveyance water lost site to surface drainage at an agricultural site depends on groundwater and evaporation loss as well as the fraction of water lost to surface drainage:

$$CNV_{da,t}^{A\_LOSS\_SW} = (W_{da,t}^{A\_AGG\_S} - RCHARG_{da,t}^{A\_CNV\_G} - CNV_{da,t}^{A\_EVAP\_LOSS}) \cdot CNV_{da}^{A\_DRNG} \tag{f.7}$$

where:
$CNV_{da,t}^{A\_LOSS\_SW}$ is the conveyance water lost to surface drainage at agriculture production site da at time t; and
$CNV_{da}^{A\_DRNG}$ is the conveyance lost to surface drainage fraction at agriculture production site da.

Relatedly, water reused from the return flow depends on the fraction of reuse as well as the conveyance water lost to surface drainage:

$$RFR_{da,t}^{A\_RUSE} = RA_{da}^{A\_DRU} \cdot CNV_{da,t}^{A\_LOSS\_SW} \tag{f.8}$$

where:
$RFR_{da,t}^{A\_RUSE}$ is the water reused from return flow at agriculture production site da at time t; and
$RA_{da}^{A\_DRU}$ is the fraction of water reuse at agriculture production site da.

Finally, water returned to the river node depends on the fraction of water returned and the return flow:

$$RFR_{da,t}^{A\_RNODE} = RA_{da}^{A\_DIVRF} \cdot (CNV_{da,t}^{A\_LOSS\_SW} - RFR_{da,t}^{A\_RUSE}) \tag{f.9}$$

where:
$RFR_{da,t}^{A\_RNODE}$ is the water returned to the river node at agriculture production site da at time t; and
$RA_{da}^{A\_DIVRF}$ is the fraction of water returned at agriculture production site da.

5.4.4 Total water available at irrigation site

The surface water delivered to an irrigation site depends on the surface water withdrawn, water reuse, and all conveyance losses:

$$\begin{aligned} W_{da,t}^{A\_DEL\_S} =& W_{da,t}^{A\_AGG\_S} + RFR_{da,t}^{A\_RUSE} - RCHRG_{da,t}^{A\_CNV\_G} - CNV_{da,t}^{A\_EVAP\_LOSS} \\ & - CNV_{da,t}^{A\_LOSS\_SW} \end{aligned} \tag{f.10}$$

where:
$W_{da,t}^{A\_DEL\_S}$ is the total surface water delivered to agricultural production site da at time t.

And the surface water actually available to crops depends on the irrigation efficiency, including efficiency gains:

$$W_{da,t}^{A\_DEL\_CRPS\_S} = W_{da,t}^{A\_DEL\_S} \cdot IRR_{da}^{A\_EFF} \cdot \left(1 + \frac{IRR_{da}^{A\_EFF\_GN}}{100}\right) \tag{f.11}$$

where:
$W_{da,t}^{A\_DEL\_CRPS\_S}$ is the surface water available for crops at agriculture production site *da* at time *t*;
$IRR_{da}^{A\_EFF}$ is irrigation efficiency at agriculture production site *da*; and
$RR_{da}^{A\_EFF\_GN}$ is irrigation efficiency gain at agriculture production site *da*.

Similarly, the groundwater actually available to crops also depends on irrigation efficiency, including efficiency gains:

$$W_{da,t}^{A\_DEL\_CRPS\_G} = W_{da,t}^{A\_DEL\_G} \cdot IRR_{da}^{A\_EFF} \cdot \left(1 + \frac{IRR_{da}^{A\_EFF\_GN}}{100}\right) \tag{f.12}$$

where:
$W_{da,t}^{A\_DEL\_CRPS\_G}$ is the groundwater available for crops at agriculture production site *da* at time *t*.

5.4.5 Total groundwater recharge

Groundwater recharge from irrigation depends on the surface and groundwater delivered as well as irrigation efficiency, including efficiency gains:

$$RCHARG_{da,t}^{A\_IRR\_G} = (W_{da,t}^{A\_DEL\_S} + W_{da,t}^{A\_AGG\_G}) \cdot \left(1 - \left(IRR_{da}^{A\_EFF} \cdot \left(1 + \frac{IRR_{da}^{A\_EFF\_GN}}{100}\right)\right)\right) \tag{f.13}$$

where:
$RCHARG_{da,t}^{A\_IRR\_G}$ is the groundwater recharge from irrigation at agriculture production site *da* at time *t*.

Total groundwater recharge is the sum of recharge from conveyance and from irrigation:

$$RCHARG_{da,t}^{A\_TOT\_G} = RCHARG_{da,t}^{A\_CNV\_G} + RCHARG_{da,t}^{A\_IRR\_G} \tag{f.14}$$

where:
$RCHARG_{da,t}^{A\_TOT\_G}$ is the total groundwater recharge at agriculture production site *da* at time *t*.

5.4.6 Return flows to WSM module

Return flow from irrigation is calculated:

$$W_{n,A,t}^{M\_RF} = \sum_{da \in NDALINK} RFR_{da,t}^{A\_RNODE} \tag{f.15}$$

where:
$W_{n,A,t}^{M\_RF}$ is the return flow from irrigation (in million m^3) at node *n* and time *t*.

Groundwater recharge from irrigation is calculated:

$$W_{n,A,t}^{M\_DF} = \sum_{da \in NDALINK} RCHARG_{da,t}^{A\_TOT\_G} \tag{f.16}$$

where:
$W^{M\_DF}_{n,A,t}$ is the groundwater recharge from irrigation (in million m^3) at node n and time t.

5.4.7 Irrigation water demand

We calculate irrigation water demand for each from using the crop coefficient, potential evapotranspiration, and taking into account effective rainfall:

$$W^{A\_DMD\_MM}_{da,c,t} = CRP^{A\_M\_COEFF}_{da,c,t} \cdot PET^{A\_DA}_{da,t} - EFF^{A\_RAIN\_DA}_{da,t} \tag{f.17}$$

where:
$W^{A\_DMD\_MM}_{da,c,t}$ is the irrigation water demand (in mm) for crop c, at agriculture production site da, at time t; and
$CRP^{A\_M\_COEFF}_{da,c,t}$ is the monthly crop coefficient for crop c, at agriculture production site da, at time t.

Then the total irrigation demand at each agriculture production site is calculated:

$$W^{A\_DMD\_SUM}_{da,t} = \sum_{c} W^{A\_DMD\_MM}_{da,c,t} \tag{f.18}$$

where:
$W^{A\_DMD\_SUM}_{da,t}$ is the total surface water irrigation demand (in million m^3) for all crops at agriculture production site da at time t.

5.4.8 Distribute water to crops

We calculate the total surface water distributed to all crops at each agricultural production site in the following way:

$$CWR^{A\_EXIST\_S}_{da,t} = \frac{W^{A\_DMD\_SUM}_{da,t}}{1000} \sum_{y \in TYLINK} AREA^{A\_IRR\_EXIST\_S}_{da,y} \tag{f.19}$$

where:
$CWR^{A\_EXIST\_S}_{da,t}$ is the total surface water irrigation distributed to crops on currently irrigated land at agriculture production site da at time t; and
$AREA^{A\_IRR\_EXIST\_S}_{da,y}$ is the total currently surface water irrigated land at agricultural production site da during year y (given the link between months and years $(y, t) \in TYLINK$) .

We allow for expansion of irrigated land in the following way:

$$CWR^{A\_EXPAND\_S}_{da,t} = \frac{W^{A\_DMD\_SUM\_S}_{da,t}}{1000} \sum_{y \in TYLINK} AREA^{A\_IRR\_EXPAND\_S}_{da,y} \tag{f.20}$$

where:
$CWR^{A\_EXPAND\_S}_{da,t}$ is the total surface water irrigation distributed to crops on potential expansion of irrigated land at agriculture production site da at time t; and
$AREA^{A\_IRR\_EXPAND\_S}_{da,y}$ is the total potentially surface water irrigable land at agricultural production

site *da* during year y (given the link between months and years $(y,t) \in TYLINK$) .

We calculate the total groundwater distributed to all crops at each agricultural production site in an identical way:

$$CWR^{A\_EXIST\_G}_{da,t} = \frac{W^{A\_DMD\_SUM}_{da,t}}{1000} \sum_{y \in TYLINK} AREA^{A\_IRR\_EXIST\_G}_{da,y} \tag{f.21}$$

where:
$CWR^{A\_EXIST\_G}_{da,t}$ is the total groundwater irrigation distributed to crops on currently irrigated land at agriculture production site *da* at time t; and
$AREA^{A\_IRR\_EXIST\_G}_{da,y}$ is the total currently groundwater irrigated land at agricultural production site *da* during year y (given the link between months and years $(y,t) \in TYLINK$) .

We allow for expansion of irrigated land in the following way:

$$CWR^{A\_EXPAND\_G}_{da,t} = \frac{W^{A\_DMD\_SUM\_S}_{da,t}}{1000} \sum_{y \in TYLINK} AREA^{A\_IRR\_EXPAND\_G}_{da,y} \tag{f.22}$$

where:
$CWR^{A\_EXPAND\_G}_{da,t}$ is the total groundwater irrigation distributed to crops on potential expansion of irrigated land at agriculture production site *da* at time t; and
$AREA^{A\_IRR\_EXPAND\_G}_{da,y}$ is the total potentially groundwater irrigable land at agricultural production site *da* during year y (given the link between months and years $(y,t) \in TYLINK$).

Then, the total surface water distributed to crops is calculated[8]:

$$W^{A\_DEL\_CRPS\_S}_{da,t,} = CWR^{A\_EXIST\_S}_{da,t} + CWR^{A\_EXPAND\_S}_{da,t} \tag{f.23}$$

and the total groundwater distributed to crops is calculated:

$$W^{A\_DEL\_CRPS\_G}_{da,t,} = CWR^{A\_EXIST\_G}_{da,t} + CWR^{A\_EXPAND\_G}_{da,t} \tag{f.24}$$

5.4.9 Agriculture production

Agriculture production from rainfed sites is calculated:

$$Q^{A\_RFD}_{da} = \sum_{y} (YLDACT^{A\_TOTAL\_RF}_{da,y} \cdot AREA^{A\_RFD}_{da,y}) \tag{f.25}$$

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[8] This characterization of water demand aggregates crop production at each agricultural site and does not allow for irrigation trade-offs between crops. Accordingly, the distribution of crops throughout the year at each agricultural site and the total productive yield associated with that distribution are critical input to the model. For a characterization of a more flexible model that does allow for within site irrigation trade-offs, see the distribution of water to specific crops in Section A.4.3 and the calculation of water deficits in Section A.4.4 (rainfed) and Section A.4.5 (irrigated). These specifications constrain the total rainfed and irrigated areas, allowing the distribution of water to vary flexibly within the model; the specifications listed here allow the irrigated areas to vary but constrain the distribution of water to crops and the cropping pattern at each agriculture production site.

where:
$Q_{da}^{A\_RFD}$ is the agriculture production (in tons) of rainfed crops across all crops at agriculture production site *da*;
$YLDACT_{da,y}^{A\_TOTAL\_RF}$ is the actual yield (in $\frac{tons}{km^2}$) of rainfed crops at agriculture production site *da* in year *y*; and
$AREA_{da,y}^{A\_RFD}$ is the total area (in km^2) used for rainfed agriculture at agriculture production site *da* in year *y*.

Agriculture production from surface water irrigated sites is calculated:

$$\begin{aligned} Q_{da}^{A\_IRR\_S} = & \sum_{y} (YLDACT_{da,y}^{A\_TOTAL\_IRR_S} \cdot AREA_{da,y}^{A\_IRR\_EXIST\_S} \\ & + YLDACT_{da,y}^{A\_TOTAL\_EXP_S} \cdot AREA_{da,y}^{A\_IRR\_EXPAND\_S}) \end{aligned} \tag{f.26}$$

where:
$Q_{da}^{A\_IRR\_S}$ is the agriculture production (in tons) of surface water irrigated crops across all crops at agriculture production site *da*;
$YLDACT_{da,y}^{A\_TOTAL\_IRR\_S}$ is the actual yield (in $\frac{tons}{km^2}$) of surface water irrigated crops on currently irrigated land at agriculture production site *da* in year *y*; and
$YLDACT_{da,y}^{A\_TOTAL\_EXP\_S}$ is the actual yield (in $\frac{tons}{km^2}$) of surface water irrigated crops on potentially irrigable land at agriculture production site *da* in year *y*.

Similarly, agriculture production from groundwater irrigated sites is calculated:

$$\begin{aligned} Q_{da}^{A\_IRR\_G} = & \sum_{y} (YLDACT_{da,y}^{A\_TOTAL\_IRR_G} \cdot AREA_{da,y}^{A\_IRR\_EXIST\_G} \\ & + YLDACT_{da,y}^{A\_TOTAL\_EXP_G} \cdot AREA_{da,y}^{A\_IRR\_EXPAND\_G}) \end{aligned} \tag{f.27}$$

where:
$Q_{da}^{A\_IRR\_G}$ is the agriculture production (in tons) of groundwater irrigated crops across all crops at agriculture production site *da*;
$YLDACT_{da,y}^{A\_TOTAL\_IRR\_G}$ is the actual yield (in $\frac{tons}{km^2}$) of groundwater irrigated crops on currently irrigated land at agriculture production site *da* in year *y*; and
$YLDACT_{da,y}^{A\_TOTAL\_EXP\_G}$ is the actual yield (in $\frac{tons}{km^2}$) of groundwater irrigated crops on potentially irrigable land at agriculture production site *da* in year *y*. Then, total crop production from irrigated sites $(Q_{da}^{A\_IRR})$ is calculated:

$$Q_{da}^{A\_IRR} = Q_{da}^{A\_IRR\_S} + Q_{da}^{A\_IRR\_G} \tag{f.28}$$

and total crop production from rainfed and irrigated sites (Q_{da}^{A}) is calculated:

$$Q_{da}^{A} = Q_{da}^{A\_RFD} + Q_{da}^{A\_IRR} \tag{f.29}$$

Total benefits depend on total crop production and prices:

$$GR_{n}^{A\_BEN} = \sum_{da \in NDALINK} Q_{da}^{A} \cdot CR_{da}^{A\_P} \tag{f.30}$$

where:
$GR_{n}^{A\_BEN}$ is the total benefit at node n; and
$CR_{da}^{A\_P}$ is the aggregated price across all crops produced at agriculture production site da.

5.4.10 Energy usage

Energy requirement in agriculture depends on energy for pumping water, delivering water, the distribution of water types used (surface, ground, reused), and the area of cropland:

$$\begin{aligned} E_{da,k,o,t}^{A\_AGG} =& IRR_{da,k,o,SWER}^{A\_CHAR} \cdot IRR_{da,k,o,SWEF}^{A\_CHAR} \cdot W_{da,t}^{A\_AGG\_S} + IRR_{da,k,o,RUER}^{A\_CHAR} \cdot IRR_{da,k,o,RUEF}^{A\_CHAR} \\ & \cdot RFR_{da,t}^{A\_REUSE} + \left(\sum_{g \in GDALINK} L_{g,k,o,t}^{E\_GPMP} \right) \cdot IRR_{da,k,o,GWEF}^{A\_CHAR} \cdot W_{da,t}^{A\_AGG} \\ & + \sum_{c} (L_{da,k,o,c,t}^{A\_APRD} \cdot AREA_{da,c}^{A\_RFD} + AREA_{da,c}^{A\_IRRSW} + AREA_{da,c}^{A\_IRRGW}) \end{aligned} \tag{f.31}$$

where:
$E_{da,k,o,t}^{A\_AGG}$ is the energy requirement in agriculture at production site da, using energy commodity o, produced using technology k, at time t;
$IRR_{da,k,o,SWER}^{A\_CHAR}$ is the energy required to deliver a unit of surface water to agriculture at production site da, using energy commodity o, produced using technology k;
$IRR_{da,k,o,SWEF}^{A\_CHAR}$ is the fraction of surface water used at agriculture production site da, using energy commodity o, produced using technology k;
$IRR_{da,k,o,RUER}^{A\_CHAR}$ is the energy required to deliver a unit of reuse water to agriculture production site da, using energy commodity o, produced using technology k;
$IRR_{da,k,o,RUEF}^{A\_CHAR}$ is the fraction of reuse water used at agriculture production site da, using energy commodity o, produced using technology k;
$L_{g,o,k,o,t}^{E\_GPMP}$ is the energy required to pump one unit of groundwater (depends on depth) from groundwater aquifer g, using energy commodity o, produced using technology k, at time t;
$IRR_{da,k,o,GWEF}^{A\_CHAR}$ is the fraction of groundwater used at agriculture production site da, using energy commodity o, produced using technology k;
$L_{da,k,o,c,t}^{A\_APRD}$ is the energy required per hectare of crops at agriculture production site da, using energy commodity o, produced using technology k, at time t;
$AREA_{da,c}^{A\_RFD}$ is the rainfed area at agriculture production site da for crop c;
$AREA_{da,c}^{A\_IRRSW}$ is the surface water irrigated area at agriculture production site da for crop c;
$AREA_{da,c}^{A\_IRRGW}$ is the groumdwater irrigated area at agriculture production site da for crop c.

Total energy withdrawn for the agricultural sector is calculated:

$$E_{n,A,k,o,t}^{M\_DIV} = \sum_{da \in NDALINK} \left(E_{da,k,o,t}^{A\_AGG} \cdot (1 + E_{da,k,o}^{A\_LOSS}) \right) \tag{f.32}$$

where:
$E_{n,A,k,o,t}^{M\_DIV}$ is the energy withdrawn at node n, for the agricultural sector A, of energy commodity o, produced using technology k, at time t; and
$E_{da,k,o}^{A\_LOSS}$ is energy loss in agriculture at agriculture production site da, using energy commodity o, produced using technology k.

5.4.11 Costs

Production costs depend on the price of energy, energy use, and other production costs such as fertilizer, labor, capital, chemical production, seeds, etc.:

$$C_{da}^{A\_PRD} = \sum_{k} \sum_{c} \sum_{t} \sum_{o \in KOLINK} \Bigg(\Big(\sum_{de \in DEDALINK} \big(P^{E}_{de,k,o,t} \cdot L_{da,k,o,c,t} \big) \Big) + V^{A\_APRD}_{da,c,t} \Bigg) \tag{f.33}$$

where:
$C_{da}^{A\_PRD}$ is production cost at agriculture production site da;
$P^{E}_{de,k,o,t}$ is the energy price at site de, for energy commodity o, produced using technology k, at time t (given the link between energy production sites and agriculture production sites $(da, de) \in DEDALINK$); and
$V^{A\_APRD}_{da,c,t}$ is other production cost at agriculture production site da, for crop c, at time t.

Water supply costs depend on the costs of water delivery by gravity, cost of surface water conveyance, costs of reuse water, costs of groundwater pumping, costs of expanding pumping capacity, and other costs:

$$\begin{aligned} C_{da}^{A\_SUP} = & \sum_{k} \sum_{t} \sum_{o \in KOLINK} \Bigg(IRR^{A\_CHAR}_{da,SWGR} \cdot (1 - IRR^{A\_CHAR}_{da,SWEF}) \cdot W^{A\_AGG\_S}_{da,t} \\ & + \Bigg(\sum_{de \in DEDALINK} P^{E}_{de,k,o,t} \Bigg) \cdot IRR^{A\_CHAR}_{da,SWER} \cdot IRR^{A\_CHAR}_{da,SWEF} + IRR^{A\_CHAR}_{da,SONC} \\ & \cdot W^{A\_AGG\_S}_{da,t} + \Bigg(\sum_{de \in DEDALINK} P^{E}_{de,k,o,t} \Bigg) \cdot IRR^{A\_CHAR}_{da,RUER} \cdot IRR^{A\_CHAR}_{da,RUEF} \\ & + IRR^{A\_CHAR}_{da,RONC} \cdot RFR^{A\_REUSE}_{da,t} + \Bigg(\sum_{de \in DEDALINK} P^{E}_{de,k,o,t} \Bigg) \\ & \cdot \Bigg(\sum_{g \in GDALINK} E\_GPMP \Bigg) \cdot IRR^{A\_CHAR}_{da,GWEF} + IRR^{A\_CHAR}_{da,GONC} \cdot W^{A\_AGG\_S}_{da,t} \Bigg) \\ & + C_{da}^{A\_PXMP\_S} + C_{da}^{A\_PXMP\_G} + C_{da}^{A\_PXMP\_R} \end{aligned} \tag{f.34}$$

where:
$C_{da}^{A\_SUP}$ is the water supply cost at agriculture production site da;
$IRR^{A\_CHAR}_{da,SWGR}$ is the fixed cost of water delivered using gravity at agriculture production site da;
$IRR^{A\_CHAR}_{da,SONC}$ is the other non-energy cost of conveying surface water at agriculture production site da;
$IRR^{A\_CHAR}_{da,RONC}$ is the other non-energy cost of conveying reuse water at agriculture production site da;
$IRR^{A\_CHAR}_{da,GONC}$ is the other non-energy cost of conveying groundwater at agriculture production site da;
$C_{da}^{A\_PXMP\_S}$ is the cost of expanding surface water pumping at agriculture production site da;
$C_{da}^{A\_PXMP\_G}$ is the cost of expanding groundwater pumping at agriculture production site da; and
$C_{da}^{A\_PXMP\_R}$ is the cost of expanding reuse water pumping at agriculture production site da.

We further calculate the cost of expanding surface water pumping as:

$$C_{da}^{A\_PXMP\_S} = \sum_{k} \sum_{o \in KOLINK} \left(IRR_{da,k,o,SPAC}^{A\_CHAR} (IRR_{da,k,o,SPCG}^{A\_CHAR})^{IRR_{da,k,o,SPBC}^{A\_CHAR}} \right) \tag{f.35}$$

where:
$IRR_{da,k,o,SPCG}^{A\_CHAR}$ is the increased surface water pumping capacity at agriculture production site da, for energy commodity o, produced using technology k; and
$IRR_{da,k,o,SPAC}^{A\_CHAR}$ and $IRR_{da,k,o,SPBC}^{A\_CHAR}$ are the parameters of non-linear regression function for the relationship between the costs and level of the surface water pumping capacity expansion at agriculture production site da, for energy commodity o, produced using technology k.

Similarly, we calculate the cost of expanding groundwater pumping as:

$$C_{da}^{A\_PXMP\_G} = \sum_{k} \sum_{o \in KOLINK} \left(IRR_{da,k,o,GPAC}^{A\_CHAR} (IRR_{da,k,o,GPCG}^{A\_CHAR})^{IRR_{da,k,o,GPBC}^{A\_CHAR}} \right) \tag{f.36}$$

where:
$IRR_{da,k,o,GPCG}^{A\_CHAR}$ is the increased groundwater pumping capacity at agriculture production site da, for energy commodity o, produced using technology k; and
$IRR_{da,k,o,GPAC}^{A\_CHAR}$ and $IRR_{da,k,o,GPBC}^{A\_CHAR}$ are the parameters of non-linear regression function for the relationship between the costs and level of the groundwater pumping capacity expansion at agriculture production site da, for energy commodity o, produced using technology k.

Finally, we calculate the cost of expanding reuse water pumping as:

$$C_{da}^{A\_PXMP\_R} = \sum_{k} \sum_{o \in KOLINK} \left(IRR_{da,k,o,RPAC}^{A\_CHAR} (IRR_{da,k,o,RPCG}^{A\_CHAR})^{IRR_{da,k,o,RPBC}^{A\_CHAR}} \right) \tag{f.37}$$

where:
$IRR_{da,k,o,RPCG}^{A\_CHAR}$ is the increased reuse water pumping capacity at agriculture production site da, for energy commodity o, produced using technology k; and
$IRR_{da,k,o,RPAC}^{A\_CHAR}$ and $IRR_{da,k,o,RPBC}^{A\_CHAR}$ are the parameters of non-linear regression function for the relationship between the costs and level of the reuse water pumping capacity expansion at agriculture production site da, for energy commodity o, produced using technology k.

The cost of improving water application efficiency depends on the cost of irrigation technology adoption and the amount of water saved:

$$\begin{aligned} C_{da}^{A\_IRR\_EFF} = & V_{da}^{A\_IRR} \cdot \left(\sum_{t} (W_{da,t}^{W\_DEL\_CRPS\_S} + W_{da,t}^{W\_DEL\_CRPS\_G}) \right) \cdot IRR_{da}^{A\_EFF} \\ & \cdot \frac{IRR_{da}^{A\_EFF\_GN}}{100} \end{aligned} \tag{f.38}$$

where:
$C_{da}^{A\_IRR\_EFF}$ is the cost of irrigation improvement at agriculture production site da; and
$V_{da}^{A_IRR}$is the cost of irrigation technology adoption per unit of water at agriculture production site

da.

The cost of improving water conveyance efficiency depends on the cost of conveyance technology adoption and the amount of water saved:

$$C_{da}^{A\_CNV\_EFF} = V_{da}^{A\_CNEF} \cdot \left(\sum_t (W_{da,t}^{A\_AGG\_S}) \right) \cdot E_{da}^{A\_CNV} \cdot \frac{E_{da}^{A\_CNV\_GN}}{100} \tag{f.39}$$

where:
$C_{da}^{A\_CNV\_EFF}$ is the cost of conveyance efficiency improvement at agriculture production site *da*; and
$V_{da}^{A\_CNEF}$ is the cost of improving conveyance efficiency per unit of water agriculture production site *da*.

5.4.12 Net benefits

We calculate the net benefit of the agricultural sector as:

$$\begin{aligned} B_{n,A}^{M\_PRD} =& GR_n^{A\_BEN} - \sum_{a \in NDALINK} (C_{da}^{A\_PRD} + C_{da}^{A\_SUP} + C_{da}^{A\_PMXP\_S} + C_{da}^{A\_PMXP\_G} \\ & + C_{da}^{A\_PMXP\_R} + C_{da}^{A\_IFF\_EFF} + C_{da}^{A\_CNV\_EFF} \end{aligned} \tag{f.40}$$

where:
$B_{n,A}^{M\_PRD}$ is the production benefit from the agricultural sector at node *n*; and
$GR_n^{A\_BEN}$ is the total gross benefit at node *n*.

5.4.13 Constraints

Surface water and reuse water pumping is constrained by the installed capacity:

$$\begin{aligned} \sum_t W_{da,t}^{A\_AGG\_S} \leq & \sum_k \sum_{o \in KOLIK} \Bigg((IRR_{da,k,o,SPCP}^{A\_CHAR} + IRR_{da,k,o,SPCG}^{A\_CHAR} + IRR_{da,k,o,RPCP}^{A\_CHAR} \\ & + IRR_{da,k,o,RPCG}^{A\_CHAR}) \cdot 3600 \cdot 24 \cdot \frac{365}{12} \Bigg) \end{aligned} \tag{f.41}$$

where: $IRR_{da,k,o,SPCP}^{A\_CHAR}$ is surface water pumping capacity ($\frac{m^3}{s}$) at agriculture production site *da*, using energy commodity *o*, produced using technology *k*;
$IRR_{da,k,o,SPCG}^{A\_CHAR}$ is surface water pumping capacity growth ($\frac{m^3}{s}$) at agriculture production site *da*, using energy commodity *o*, produced using technology *k*;
$IRR_{da,k,o,RPCP}^{A\_CHAR}$ is reuse water pumping capacity ($\frac{m^3}{s}$) at agriculture production site *da*, using energy commodity *o*, produced using technology *k*; and
$IRR_{da,k,o,RPCG}^{A\_CHAR}$ is reuse water pumping capacity growth ($\frac{m^3}{s}$) at agriculture production site *da*, using energy commodity *o*, produced using technology *k*.

Similarly, groundwater pumping is constrained by the installed capacity:

$$\sum_{t} W^{A\_AGG\_G}_{da,t} \leq \sum_{k} \sum_{o \in KOLIK} \left(\left(IRR^{A\_CHAR}_{da,k,o,GPCP} + IRR^{A\_CHAR}_{da,k,o,GPCG} \right) \cdot 3600 \cdot 24 \cdot \frac{365}{12} \right) \tag{f.42}$$

where: $IRR^{A\_CHAR}_{da,k,o,GPCP}$ is groundwater pumping capacity $(\frac{m^3}{s})$ at agriculture production site da, using energy commodity o, produced using technology k; and
$IRR^{A\_CHAR}_{da,k,o,GPCG}$ is groundwater pumping capacity growth $(\frac{m^3}{s})$ at agriculture production site da, using energy commodity o, produced using technology k.

The upper bound of land for rainfed agriculture is defined as the land at each agriculture production site currently used for rainfed agriculture $(AREA^{A\_TOTAL\_RFD}_{da,y})$:

$$AREA^{A\_RFD\_.up}_{da,y} = AREA^{A\_TOTAL\_RFD}_{da,y} \tag{f.43}$$

The land for existing surface water and groundwater irrigated agriculture is constrained by the land at each agriculture production site currently irrigated $(AREA^{A\_TOTAL\_IRR}_{da,y})$:

$$AREA^{A\_TOTAL\_IRR}_{da,y} \geq AREA^{A\_IRR\_EXIST\_S}_{da,y} + AREA^{A\_IRR\_EXIST\_G}_{da,y} \tag{f.44}$$

Similarly, the land for surface water and groundwater irrigation expansion is constrained by the potentially irrigable land at each agriculture production site $(AREA^{A\_POTENTAIL\_IRR}_{da,y})$:

$$AREA^{A\_POTENTIAL\_IRR}_{da,y} \geq AREA^{A\_IRR\_EXPAND\_S}_{da,y} + AREA^{A\_IRR\_EXPAND\_G}_{da,y} \tag{f.45}$$

Finally, the area used for agriculture at any production site is constrained by the total cultivable land at that site $(AREA^{A\_TOTAL\_CUL}_{da,y})$:

$$\begin{aligned} AREA^{A\_TOTAL\_CUL}_{da,y} \geq & AREA^{A\_TOTAL\_RFD}_{da,y} + AREA^{A\_IRR\_EXIST\_S}_{da,y} + AREA^{A\_IRR\_EXIST\_G}_{da,y} \\ & + AREA^{A\_IRR\_EXPAND\_S}_{da,y} + AREA^{A\_IRR\_EXPAND\_G}_{da,y} \end{aligned} \tag{f.46}$$

5.5 Environmental module

5.5.1 Water balance

Water flow at a particular node n that is available for downstream flow is calculated as the sum of all associated upstream water flows:

$$FLOW^{G\_DS}_{n,t} = \sum_{nd \in NNDLINK} W^{W\_F}_{n,nd,t} \tag{g.1}$$

where:
$FLOW^{G\_DS}_{n,t}$ is the flow at node n and time t available for downstream flow;
$W^{W\_F}_{n,nd,t}$ is the flow from upstream at node N and time t (given the link between nodes $(n, nd) \in NNDLINK$).

5.5.2 Benefits

Ecosystem benefits $(GROSS^{G\_BEN}_{n,es,t})$ depend on a set of parameters used to calculate the benefits as well as the downstream flow:

$$\begin{aligned} GROSS^{G\_BEN}_{n,es,t} = & ESS^{G\_PARMS}_{n,es,A} (FLOW^{G\_DS}_{n,t})^{ESS^{G\_PARMS}_{n,es,B}} + ESS^{G\_PARMS}_{n,es,C} (FLOW^{G\_DS}_{n,t})^{ESS^{G\_PARMS}_{n,es,D}} \\ & + ESS^{G\_PARMS}_{n,es,E} \end{aligned} \tag{g.2}$$

where:
$ESS^{G\_PARMS}_{n,es,A}$, $ESS^{G\_PARMS}_{n,es,B}$, $ESS^{G\_PARMS}_{n,es,C}$, $ESS^{G\_PARMS}_{n,es,D}$, and $ESS^{G\_PARMS}_{n,es,E}$ are parameters used in ecosystem functions at node n for ecosystem service es.

The net benefits of ecosystem services $(B^{M\_PRD}_{n,Env})$ depend on the gross benefit and the cost $(ESS^{G\_COST}_{n,es})$:

$$B^{M\_PRD}_{n,Env} = \sum_{es} \sum_{t} GROSS^{G\_BEN}_{n,es,t} - \sum_{es} ESS^{G\_COST}_{n,es} \tag{g.3}$$

5.5.3 Constraints

Environment flows $(ENVFLOW^{G}_{n,t})$ are constrained according to:

$$FLOW^{G\_DS}_{n,t} \geq ENVFLOW^{G}_{n,t} \tag{g.4}$$

6 Application

This HEM was first applied to the Karnali and Mahakali River Basins (see Figure 16), which span nearly 47,000 square kilometers in Western Nepal (for more complete details of this application as well as the results, see Pakhtigian and Jeuland (2019a)). Like the rest of Nepal, the Karnali and Mahakali River Basins are characterized by river resources that are vast in terms of potential–particularly for hydropower generation–yet largely undisturbed.[9] Furthermore, the economy of Western Nepal is dominated by agriculture, and Nepal's unique and valuable natural ecosystems have brought environmental conservation to the forefront of development planning among some key stakeholders in water resource development (Pakhtigian et al., 2019). These characteristics make Western Nepal an ideal context for the application of a HEM based on the WEEF nexus, which seeks to capture the integration of water resource use across energy, agriculture, and environmental sectors.

The Western Nepal application follows the structure of the model outlined above to optimize water resource use across energy, agriculture, municipal, and environmental demands. While we focus primarily on water use within Nepal, we acknowledge that transboundary considerations, particularly in the Mahakali River Basin as the Mahakali River forms the boundary between Nepal and India, enter into the model in two distinctive ways–through downstream water requirements and through energy export. These and other considerations are also explored more thorough in sensitivity analyses (Pakhtigian and Jeuland, 2019a).

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[9] The Karnali and Mahakali Rivers have an estimated hydropower generation potential of around 35,000 MW (Sharma and Awal, 2013), yet installed capacity remains around 10 MW with no storage infrastructure existing across the basins.

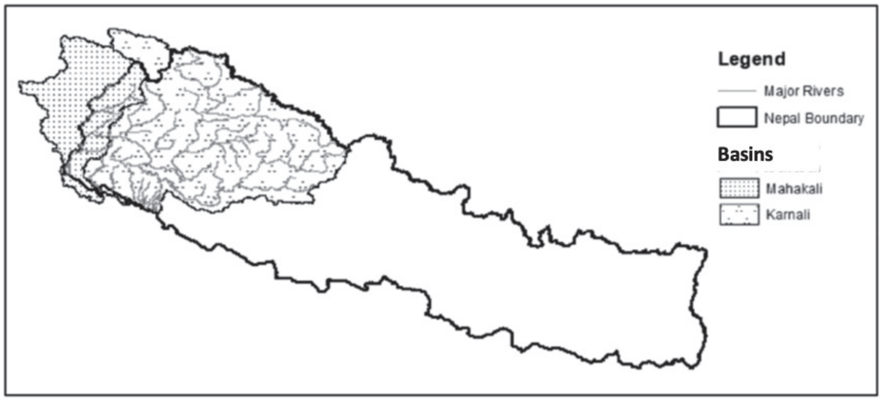

Figure 16: Map of the Karnali and Mahakali River Basins, Western Nepal.

The model structure is maintained by a set of nodes that are connected by flows links, which reflect the hydrology, municipal demands, energy production, and agricultural production throughout the system (see Figure 17). The model comprises 151 river nodes. Additionally, there are 55 energy production nodes, which identify existing, planned, or proposed run-of-the-river or storage hydropower projects, and 37 agricultural production nodes, which identify existing, planned, or under construction irrigation projects. Municipal surface water demands are satisfied at each of the 151 river nodes, as are environmental flow constraints. The model is run using hydrology that spans a period of 12 years, with different combinations of infrastructure. Specification of production, biophysical, and economic relationships relies on a variety of data sources.

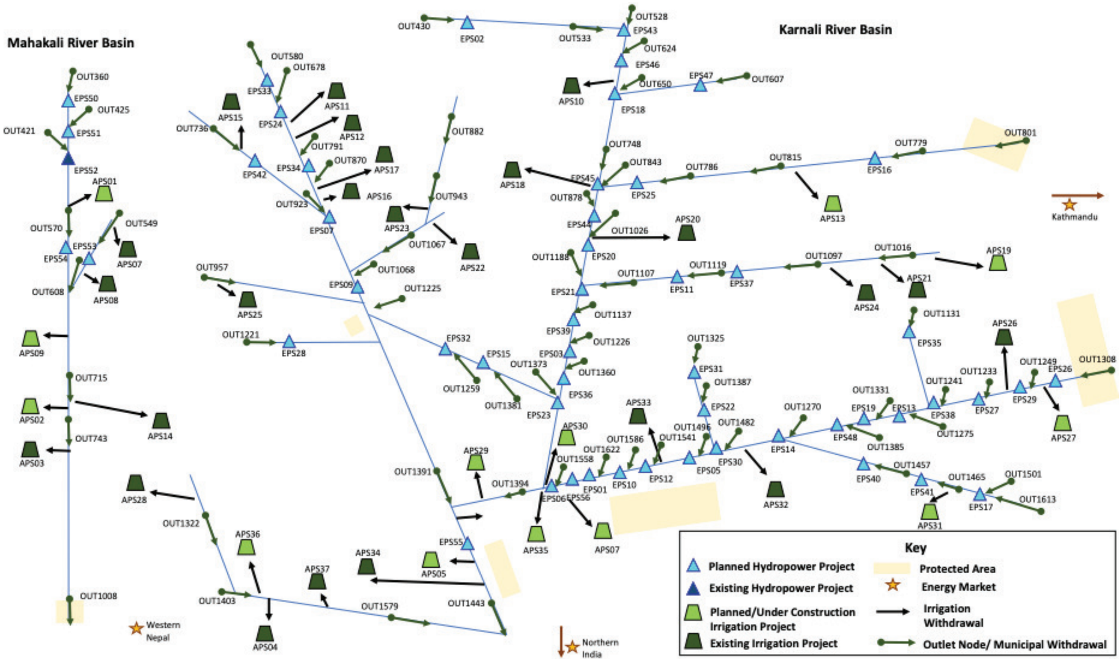

Figure 17: Schematic of the node system used in the Western Nepal application.

6.1 Data for model

The HEM is data intensive, as application-specific parameters are required to ensure that the model accurately reflects operations in the river basins under consideration. Here, we briefly outline the main data sources and tools used to parameterize the model; more details regarding each data source are provided in Pakhtigian and Jeuland (2019a). The hydrological data inputs are generated from a ArcSWAT model developed for the region and described in (Pandey et al., 2019). These data include source flow generated at each of the river nodes as well as precipitation and evaporation.

The energy module, which in this application focuses exclusively on electricity generated via hydropower, is parameterized using data from hydropower reports and, when available, project-specific documentation. In particular, energy production sites were determined based on existing licenses for projects above 1 MW granted by the Government of Nepal. The existence of a license does not guarantee that a project exists, is under construction, or has financing; rather, projects are separated into four categories: existing, under construction, planned, and proposed. Many of these project sites and capacities are mentioned or detailed in government-commissioned reports such as the Hydropower Development Plan, the Master Plan Study for Water Resource Development of Upper Karnali and Mahakali River Basin, and the Nationwide Master Plan Study on Storage-type Hydroelectric Power Development in Nepal. In addition, for projects under planning or construction phases, project-specific documents outlining more specific dimensions of the project, particularly reservoir characteristics for planned and proposed storage projects. Finally, reports from the Nepal Electricity Authority provided details about electricity prices, costs, transmission, and efficiency.

A combination of modeling and reports formed the basis for parameterization of the agriculture module. The set of agriculture production sites was established based on existing agriculture as well as existing, under construction, planned, or proposed irrigation infrastructure sites. These lists were available in documents such as the Nepal Department of Irrigation's Irrigation Master Plan, the National Irrigation Database, and communications with personnel at the the Department of Irrigation. The CROPWAT and CLIMWAT tools developed by the Food and Agricultural Organization (FAO) were used to calculate crop water requirements, evapotranspiration, and crop coefficients. In addition the annual Statistical Information on Nepalese Agriculture and other Ministry of Agriculture documents provided parameters regarding cropping patterns, crop prices and costs, and irrigation practices.

Municipal water and energy demands were calculated based on information from Water User Master Reports, national statistics, and data from a household-level survey (see Pakhtigian and Jeuland (2019b) for a description of the household-level survey data). Finally, environmental flows (e-flows) were calculated using an environmental flows calculator developed for Western Nepal. These e-flows also capture cultural demands on river resources.

6.2 Model simplifications and deviations

Data availability and context-specific characteristics of Western Nepal require certain model simplifications and deviations from the general model outlined in the previous section. In the next five subsections, we clarify and explain these deviations.

6.2.1 Hydrology core

Throughout the generalized model, both in the hydrology core and in other sector modules, water is separated into two categories based on source–surface water and groundwater. In our application, we model only surface water. The primary reason for this deviation is the lack of comprehensive groundwater data for Western Nepal, making it infeasible to incorporate groundwater access, demands, and use into the HEM application. There are two primary concerns associated with this omission. First, there is evidence of some trade-offs in municipal water use between surface and ground water which our model is unable to capture in this application. That is, households cannot supplement decreases in surface water access with groundwater (or vice versa) due to the lack of groundwater data. These trade-offs are likely concentrated in the Terai–the southern plain region–suggesting that the lack of groundwater to supplement surface water access in the model would provide a conservative estimate for overall productive benefits because the main trade-off region exists downstream. Second, we are unable to account for expansion of groundwater irrigation. While currently there are few large-scale groundwater irrigation schemes in the region, it is possible that expansion of groundwater irrigation would provide a viable water source for farmers in the Terai, and this is missing from our application.

Reservoir relationships, in our application used exclusively in conjunction with storage hydropower projects, are also calculated as part of the core hydrology module. In the Western Nepal application, we impose linear relationships between area and volume (and net head and volume) rather than the polynomial relationships specified in the general model. Again, data limitations regarding the exact site location of reservoirs, force this simplification. These linear relationships will also provide a conservative estimate of reservoir volume, and, subsequently, energy generation.

6.2.2 Energy

The main simplification in the energy module relates to the specific energy context in Western Nepal. Nepal has vast river resources and hydropower potential; with investments in storage infrastructure to regulate water availability, Nepal's hydropower potential could meet domestic electricity demands and form a basis of energy export trade with neighboring countries, particularly India. Accordingly, in this application we focus on just one energy generating technology–hydropower–and just one energy commodity–electricity. Given this technology, water requirements for energy generation are tied directly to water availability at a river node (for an energy production site with run-of-the-river infrastructure) or a reservoir node (for an energy production site with storage infrastructure). Furthermore, demands for energy commodities as inputs to energy generation via hydropower are minimal.

6.2.3 Industrial and Municipal

The Western Nepal application does not include industrial water demands. There is very limited industrial production in Western Nepal and, while eco-tourism and environmental conservation do provide one potential avenue of development in the region (Pakhtigian et al., 2019), recreational and hospitality demands on water resources are captured within municipal water demands and e-flow constraints. Thus, the context of this application is ill-suited to incorporate industrial water demands as part of the model. Relatedly, wastewater treatment are uncommon throughout Western Nepal, so this component is omitted from the application.

For the municipal sector, we apply municipal water constraints rather than incorporating values associated with provision of water to meet municipal demands. Water resource stakeholders at both national and local levels recognize the importance of surface water resources to meet municipal water demands and often prioritize municipal access over water uses for productive sectors like agriculture and energy if a proposed infrastructure project would incur such a trade-off (Pakhtigian et al., 2019). Accordingly, we constrain diversions to ensure some level of surface water access at each river node based on the population surrounding each river node and demands for surface water in each geographically distinct portion of the region (i.e., the demands for surface water to meet municipal needs are different in the mountains compared to the mid-hills or Terai). With regard to municipal energy demands, while in our main specification we allow energy to flow to markets where it is most beneficial from an economic perspective, we do calculate how much of municipal energy demand in met in each specification and conduct sensitivity analyses which constrain the distribution of energy across energy markets in alternative ways.

6.2.4 Agriculture

Within the agriculture model, our application follows closely to the equations specified in the section above. Importantly, as the model allows cultivated areas (both rainfed and irrigated) to vary as it solves, the cropping patterns, pricing, and cost data are aggregated to the agricultural production site-year level. Furthermore, while there is likely variation within-district regarding crop prices, agricultural costs, and other parameters within the agriculture module, district-level data was the finest resolution available; thus, the model does not incorporate intra-district variation across agricultural parameters. The main data limitations in agriculture include a lack of information on energy demands in agriculture, particularly those related to water pumping for irrigation. Finally, water reuse in agriculture in uncommon in Western Nepal, so water reuse is omitted from the model.

6.2.5 Environmental

As with the municipal module, we incorporate the environmental sector as a system of constraints rather than ascribing value to ecosystem-related water use. In particular, we incorporate a variety of e-flow constraints that allow for different levels of diversions from the river. These e-flows are calculated to maintain aquatic integrity in the rivers. Thus, the difference between model outcomes in the presence and absence of the e-flow constraints provides insight into the economic value the existence of aquatic ecosystems or maintenance of river flows for recreation, navigation, or other purposes would need to afford to promote the binding of these e-flow constraints.

7 Discussion and Summary

Increasing competition for water resources among multiple economic and social sectors calls for efficient allocation of water and intelligent trade-offs among sectors. These, in turn, require a planning approach that incorporates development trajectories and portfolios of management and investment solutions. To support such an integrated planning approach there is a need for tools that better account for the complex social and physical dynamics underlying water systems. This report described an HEM structure that is based around the concept of the Water-Energy-Environment-Food (WEEF) Nexus. The specific structure of the HEM has been developed to describe the integrated social-physical system with three core principles in mind: scalability, transferability, and modularity.

The first two principles allow the model to be implemented in any catchment or river basin with minimal changes. The third principle allows the model to be more effective in handling research questions by turning "on" and "off" relevant modules based on the research question at hand.

More specifically, our HEM Nexus framework depicts interactions between five specific sectors or modules. The first core module, which contains the model objective function, is the water system. It is based on the typical node-link structure of most similar HEMs. This module also includes surface and groundwater interlinkages as appropriate. This objective function aims for maximization of benefits across sectors and uses given both physical and social water and energy system relationships and constraints. Three other modules that are linked to this core are principally human production systems; these represent the energy, municipal and industrial, and agricultural production systems, organized around the representation of the water system core. A fifth module describes the broader ecosystem or environment; this component provides a variety of market and nonmarket goods and services (ecosystem services) to the other systems and is also the recipient of "externalities" from these systems. These externalities, beyond certain levels, may lead to a reduction in the ability of ecosystem to provide services to other systems and to the broader environment.

This model forms an important component for a Decision Support System (DSS). It must be linked to a database of parameters for use in the model equations. Following the model parameterization, users can explore efficient water allocations and specify scenarios or changes to the system that would affect those efficient solutions. Given the inherent complexity in integrated water resources systems, such scenario analyses can help provide more reliable, or data-driven, understanding of the potential costs and benefits of policy and investment changes across multiple sectors that are linked to a water resources system. They can also illuminate critical policy trade-offs and their implications for users or interests in different locations.

As an optimization model, the HEM Nexus tool is well-adapted to identifying solutions that most efficiently allocate water and other resources, which is especially useful for planning purposes at the basin level. As with all similar models, these work from a standardized and simplified representation of very complex system that is developed to be both sufficiently realistic and computationally tractable. Such models are sometimes criticized for the assumptions inherent in their structure. Optimization frameworks in particular may not be well-suited to understanding real world outcomes because the institutions governing allocations rarely come close to resembling an omniscient social planner or a well-functioning water market. In addition, the model is not meant to be used for operational purposes, which typically require greater spatial and temporal resolution. Finally, the HEM Nexus described here is new, and needs to be applied to a variety of problems and contexts to improve its usability and relevance to real world situations, and to better streamline the nature of its data requirements.

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A Potential Future Extensions

Extensions to the contained modules are included as potential expansions of the model subject to appropriate data availability.

A.1 Water Module

A.1.1 Two-way surface and groundwater flows

For any particular node, there cannot be both seepage into groundwater (from the river) and seepage out of groundwater (into the river) in the same month.

$$W_{g,t}^{W\_GWS} \cdot W_{g,t}^{W\_GWC} = 0 \tag{Aw.1}$$

In the water module, the physical limitations of the aquifer are accounted for by the inclusion of a maximum groundwater level constraint (Equation w.10). Thus, the actual pumping head can be specified as:

$$Z_{g,t} = AQ_{g,MXH}^{B\_CHAR} - GW_{g,t}^{W\_D.lo} + ghd0_{g,t}^{W} \tag{Aw.2}$$

where:
$AQ_{g,MXH}^{B\_CHAR}$ is the height at the top of groundwater aquifer g;
$GW_{g,t}^{W\_D.lo}$ is the head of groundwater aquifer g at time t; and
$ghd0_{g,t}^{W}$ is the pump draw-down of groundwater aquifer g at time t.

Groundwater seepage into surface river flow depends on the water volume in the groundwater aquifer and the transitivity coefficient $(\varphi_{g,n})$

$$W_{g,t}^{W\_GWS} = 0.01 \cdot \varphi_{g,n} G_{g}^{W\_YGW0} A_{g}^{W\_GWA0} \left(\frac{H_{g,t}^{W\_GWA} + H_{g,t-1}^{W\_GWA}}{2} \right) \tag{Aw.3}$$

Aquifer recharge through river flows are considered through linear relationship between the amount of the recharge and river flow:

$$W_{d,t}^{W\_GWC} = \sum_{g \in NGLINK} \left(r_{n,t}^{W\_RGW} \sum_{nu \in NNULINK} W_{nu,n,t}^{W\_F} \right) \tag{Aw.4}$$

where:
$r_{n,t}^{W\_RGW}$ is the share of river flow to charge a groundwater aquifer g at node n and time t (given the link between groundwater aquifers and nodes $(g, n) \in NGLINK$).

A.2 Energy Module

A.2.1 Endogenous quantity adjustments

Given time-varying price data, energy supply in market m depends on the price of energy commodity o:

$$L_{m,o,t}^{E\_SUP} = \alpha_{m,o,t}^{E\_END} (P_{m,o,t}^{E\_M})^{\beta_{m,o,t}^{E\_END}} \tag{Ae.1}$$

where:
$\alpha^{E\_END}_{m,o,t}$ and $\beta^{E\_END}_{m,o,t}$ are the parameters of the exponential regression function; and
$P^{E\_M}_{m,o,t}$ is the price for energy commodity o at market m.

Energy prices for commodity o used at production site de and in its related regional energy market are the same:

$$P^{E}_{de,o,t} = \sum_{m \in MELINK} P^{E\_M}_{m,o,t} \tag{Ae.2}$$

A.3 Industrial/Municipal Module

A.3.1 Leontief production

The relationship between value added by industrial sector and uses of water and energy resources is considered to be a Leontief production process:

$$\frac{VA^{I}_{di}}{\bar{V}A^{I\_0}_{di}} \leq \left(\frac{\sum_t W^{I\_USE}_{di,t}}{\sum_t \bar{W}^{I\_USE0}_{di,t}} \right) \tag{Ai.1a}$$

$$\frac{VA^{I}_{di}}{\bar{V}A^{I\_0}_{di}} \leq \left(\frac{\sum_t \sum_o (f^{I\_O}_{di,o} L^{I\_PRD}_{di,o,k,t})}{\sum_t \sum_o (f^{I\_O}_{di,o} \bar{L}^{I\_PRD}_{di,o,k,t})} \right) \tag{Ai.1b}$$

where:
VA^{I}_{di} and $\bar{V}A^{I}_{di}$ are actual and baseline industrial value added at industrial site di;
$W^{I\_USE}_{di,t}$ and $\bar{W}^{I\_USE0}_{di,t}$ are actual and baseline water uses at industrial site di at time t;
$L^{I\_PRD}_{di,o,k,t}$ and $\bar{L}^{I\_PRD}_{di,o,k,t}$ are actual and baseline energy uses at industrial site di, using energy commodity o, produced by technology k, at time t; and
$f^{I\_O}_{di,o}$ is a weight factor used to make electricity and diesel use units comparable.

This production function is based on an assumption of no substitution between water and energy resources in industrial production but allows substitution between electricity and diesel.

A.4 Agriculture Module

A.4.1 Calculating effective rainfall

In the event that data on effective rainfall is unavailable, it can be calculated. To calculate effective rainfall, run the following loop over every node n:

$$EFF^{A\_RAIN}_{n,t} = 0 \text{ if } PPT^{W}_{n,t} \leq 10 \tag{Af.1a}$$

$$EFF^{A\_RAIN}_{n,t} = 0.2 \cdot (PPT^{W}_{n,t} - 10) \text{ if } 10 < PPT^{W}_{n,t} \leq 20 \tag{Af.1b}$$

$$EFF^{A\_RAIN}_{n,t} = 2 + 0.6 \cdot (PPT^{W}_{n,t} - 20) \text{ if } 20 < PPT^{W}_{n,t} \leq 70 \tag{Af.1c}$$

$$EFF^{A\_RAIN}_{n,t} = 32 + 0.7 \cdot (PPT^{W}_{n,t} - 70) \text{ if } 70 < PPT^{W}_{n,t} \leq 80 \tag{Af.1d}$$

$$EFF^{A\_RAIN}_{n,t} = 39 + 0.8 \cdot (PPT^{W}_{n,t} - 70) \text{ if } PPT^{W}_{n,t} > 80 \tag{Af.1e}$$

A.4.2 Endogenous quantity adjustments

The total amount of the crop produced in the basin depends on crop prices:

$$\sum_{da} Q_{a,c}^{A\_CRP} = \alpha_c^{A\_AGD} (P_c^A)^{\beta_c^{A\_AGD}} \tag{Af.2}$$

where:
$\alpha_c^{A\_AGD}$ and $\beta_c^{A\_AGD}$ are the coefficients of the agricultural commodity demand function that relate crop price to the produced amount.

A.4.3 Distribute water to crops

We define $W_{da,t}^{A\_SUM\_P}$ in the following way as a placeholder for the product of water stress, area, and crop price:

$$W_{da,t}^{A\_SUM\_P} = \sum_c (CRP_{da,c,t}^{A\_WS\_COEFF} \cdot AREA_{da,c}^{A\_IRRSW} \cdot CR_c^{A\_P}) \tag{Af.3}$$

where:
$CRP_{da,c,t}^{A\_WS\_COEFF}$ is the water stress coefficient for agriculture production site *da*, crop *c*, at time *t*; and
$CR_c^{A\_P}$ is the crop price.

Then the surface water available for each crop $(CR_{da,c,t,}^{A\_AVB\_VOL\_S})$ (in million m^3) is calculated:

$$CR_{da,c,t}^{A\_AVB\_VOL\_S} = W_{da,t}^{A\_DEL\_CRPS\_S} \cdot \left(CRP_{da,c,t}^{A\_WS\_COEFF} \cdot AREA_{da,c}^{A\_IRRSW} \cdot \frac{CR_c^{A\_P}}{W_{da,t}^{A\_SUM\_P}} \right) \tag{Af.4}$$

Similarly, the groundwater available for each crop $(CR_{da,c,t,}^{A\_AVB\_VOL\_G})$ (in million m^3) is calculated:

$$CR_{da,c,t}^{A\_AVB\_VOL\_G} = W_{da,t}^{A\_DEL\_CRPS\_G} \cdot \left(CRP_{da,c,t}^{A\_WS\_COEFF} \cdot AREA_{da,c}^{A\_IRRSW} \cdot \frac{CR_c^{A\_P}}{W_{da,t}^{A\_SUM\_P}} \right) \tag{Af.5}$$

We convert these to surface water available in mm:

$$CR_{da,c,t}^{A\_AVB\_MM\_S} = \frac{CR_{da,c,t}^{A\_AVB\_VOL\_S}}{AREA_{da,c}^{A\_IRRSW}} \cdot 1000 \tag{Af.6}$$

And groundwater available to each crop in mm:

$$CR_{da,c,t}^{A\_AVB\_MM\_G} = \frac{CR_{da,c,t}^{A\_AVB\_VOL\_G}}{AREA_{da,c}^{A\_IRRGW}} \cdot 1000 \tag{Af.7}$$

A.4.4 Rainfed crops: Calculate deficit

The stage deficit for rainfed crops ($D^{A\_R}_{da,c,t}$) depends on the effective rainfall, water available through irrigation, the crop coefficient, and the potential evapotranspiration. Here, only crops with a positive crop coefficient, those exhibiting water demand, are included in the calculation. The stage deficit is calculated:

$$D^{A\_R}_{da,c,t} = 1 - \left(\frac{EFF^{A\_RAIN\_DA}_{da,t}}{CRP^{A\_M\_COEFF}_{da,c,t} \cdot PET^{A\_DA}_{da,t}} \right) \tag{Af.8}$$

The maximum stage deficit ($DMAX^{A\_R}_{da,c}$) is, in turn, estimated based on monthly stage deficits:

$$DMAX^{A\_R}_{da,c} = \max_t (D^{A\_R}_{da,c,t}) \tag{Af.9}$$

Next, we calculate seasonal relative yield for rainfed crops ($YLDS^{A\_REL\_R}_{da,c}$), which depends on effective rainfall, the seasonal crop coefficient, and potential evapotranspiration:

$$YLDS^{A\_REL\_R}_{da,c} = \frac{\sum_t EFF^{A\_RAIN\_DA}_{da,t}}{\sum_t (CRP^{A\_S\_COEFF}_{da,c,} \cdot PET^{A\_DA}_{da,t})} \tag{Af.10}$$

where:
$CRP^{A\_S\_COEFF}_{da,c,}$ is the seasonal crop coefficient at agriculture production site *da* specific to crop *c*.

The minimum relative yield for rainfed crops ($YLDREL^{A\_MIN\_R}_{da,c}$) is calculated:

$$YLDREL^{A\_MIN\_R}_{da,c} = \min(1 - DMAX^{A\_R}_{da.c}, YLDS^{A\_REL\_R}_{da,c}) \tag{Af.11}$$

Finally, the actual yields ($YLDACT^{A\_R}_{da,c}$) depend on relative and potential yields:

$$YLDACT^{A\_R}_{da,c} = YLDREL^{A\_MIN\_R}_{da,c} \cdot RFD^{A\_P\_YLD}_{da,c} \tag{Af.12}$$

where: $RFD^{A\_P\_YLD}_{da,c}$ is the potential rainfed yield of crop *c* at agriculture production site *da*.

A.4.5 Irrigated crops: Calculate deficit

In calculating the stage deficit for irrigated crops, we recognize two different water sources for irrigation–surface water and ground water. As the deficit is calculated the same way for both sources, we let $S, G \in \nu$. Then the stage deficit for irrigated crops ($D^{A\_I\_\nu}_{da,c,t}$), where ν indicates either surface or groundwater, depends on effective rainfall, water available for each crop, the monthly crop coefficient, and potential evapotranspiration. As with rainfed crops, only crops with a positive crop coefficient, those exhibiting water demand, are included in the calculation. The stage deficit is calculated:

$$D^{A\_I\_\nu}_{da,c,t} = 1 - \left(\frac{EFF^{A\_RAIN\_DA}_{da,t} + CR^{A\_AVB\_MM\_\nu}_{da,c,t}}{CRP^{A\_M\_COEFF}_{da,c,t} \cdot PET^{A\_DA}_{da,t}} \right) \tag{Af.13}$$

The maximum deficit among stages for irrigated crops ($DMAX^{A\_I\_v}_{da,c}$) is, in turn, estimated based on monthly stage deficits:

$$DMAX^{A\_I\_v}_{da,c} = \max_t (D^{A\_I\_v}_{da,c,t}) \tag{Af.14}$$

Next, we calculate seasonal relative yield for irrigated crops ($YLDS^{A\_REL\_I\_v}_{da,c}$), which depends on effective rainfall, available irrigation, the seasonal crop coefficient, and potential evapotranspiration:

$$YLDS^{A\_REL\_I\_v}_{da,c} = \frac{\sum_t (EFF^{A\_RAIN\_DA}_{da,t} + CR^{A\_AVB\_MM\_v}_{da,c,t})}{\sum_t (CRP^{A\_S\_COEFF}_{da,c,} \cdot PET^{A\_DA}_{da,t})} \tag{Af.15}$$

The minimum relative yield for irrigated crops ($YLDREL^{A\_MIN\_I\_v}_{da,c}$) is calculated:

$$YLDREL^{A\_MIN\_I\_v}_{da,c} = \min(1 - DMAX^{A\_I\_v}_{da,c}, YLDS^{A\_REL\_I\_v}_{da,c}) \tag{Af.16}$$

Finally, the actual yields for irrigated crops ($YLDACT^{A\_I\_v}_{da,c}$) depend on relative and potential yields:

$$YLDACT^{A\_I\_v}_{da,c} = YLDREL^{A\_MIN\_I\_v}_{da,c} \cdot IRR^{A\_P\_YLD}_{da,c} \tag{Af.17}$$

where:
$IRR^{A\_P\_YLD}_{da,c}$ is the potential irrigated yield of crop c at agriculture production site da.